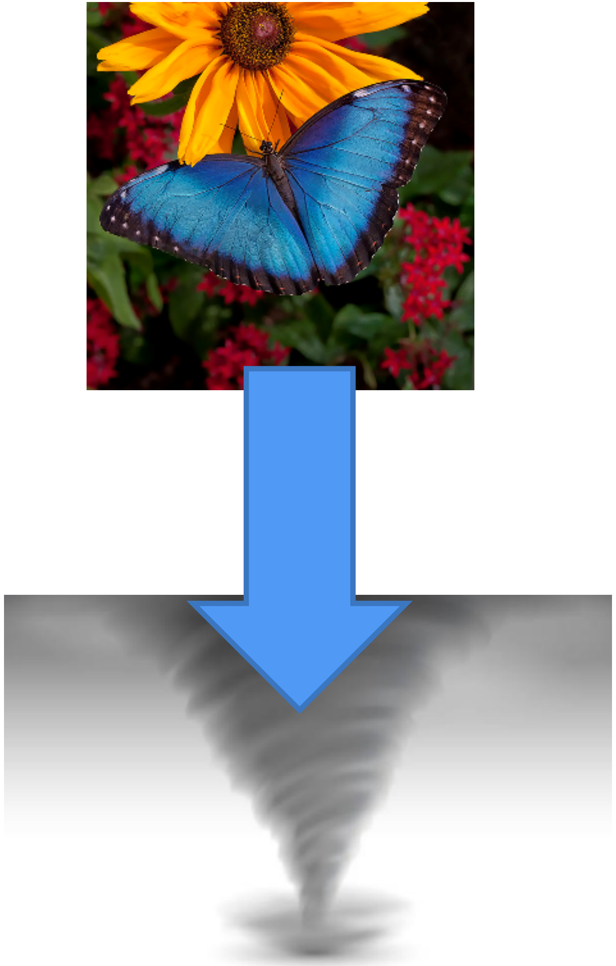
Order out of chaos

Grade 4

[mathematics, computer science]

Background

- Chaos theory is the study of how order can arise out of chaos.
- You might think that there's no way to predict chaotic events.
- Think of a ball that rolls down a hill. What path will it take? It seems impossible to predict. But chaos theory says we can sometimes see order resulting from chaos.
- Chaos theory studies patterns in chaos that lead to order. Probably the best known idea in chaos theory is the butterfly effect: a butterfly flapping its wings in Brazil could cause a tornado in Texas. Physicists, mathematicians, and biologists study chaos.

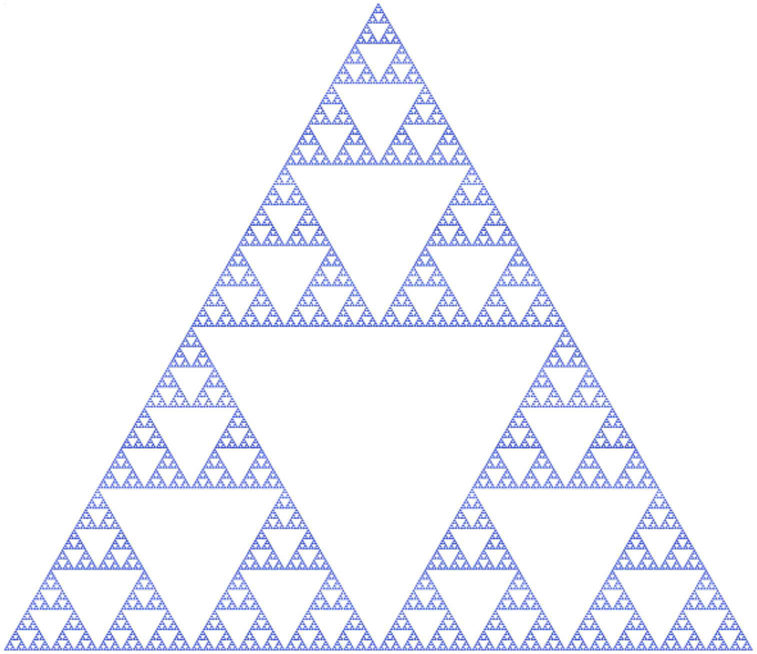


Background

- Here we look at the concept of an **attractor**.
- An attractor is a situation that a system will evolve towards.
- Think of a fern leaf. Though it can look many different ways, it will always end up growing in a fractal pattern.
- Fractals are patterns that never end, and where part of the pattern resembles the whole.
- We see lots of fractals in nature. An example in your kitchen is a romanesco broccoli: a little piece of it (a floret) resembles the whole!
- Benoit Mandelbrot described these in his book *The Fractal Geometry of Nature* (1982).

Testable hypothesis

- How do we make a fractal pattern? Mandelbrot (1982) said that they are made through mathematical rules.
- An example is the chaos game, described here https://en.wikipedia.org/wiki/Sierpi%C5%84ski_triangle#Chaos_game
- To the right you see the Sierpinski triangle, which is a fractal pattern. As you can see, it replicates itself in smaller and smaller scales.
- I will examine whether I can replicate the chaos game on the computer using the programming language Scratch (MIT).



Procedure

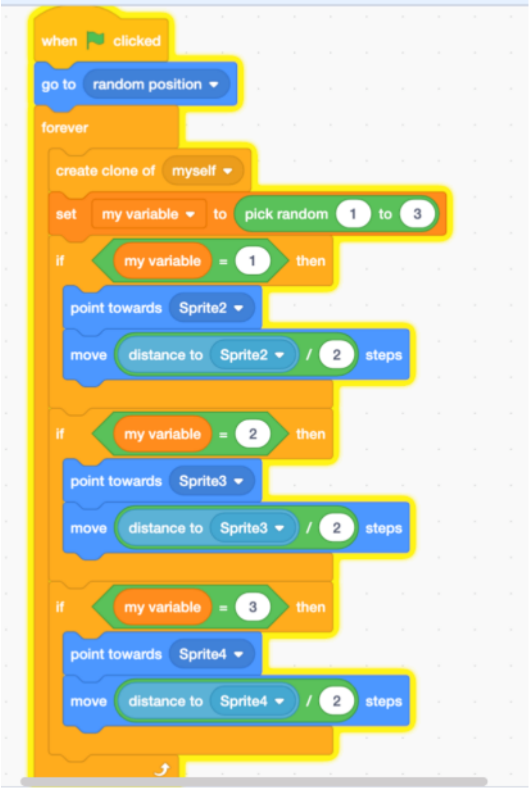
The triangle chaos game works as follows:
1. draw three points in the shape of any triangle. These will be the vertices of the triangle.
2. draw a point somewhere in proximity of one of the vertices. (It doesn't even need to be inside the triangle.)
3. the rules are as follows:
"from the last point you drew, advance randomly to one of the vertices of the triangle in a straight line. The distance you advance is half the distance between that point and the vertex."

Continue doing this, and a Sierpinski triangle will form.

The mathematical version of this set of rules is: for a randomly picked point v1, and vertices of the triangle p1, p2, p3, set $\mathbf{v}_{n+1} = 1/2(\mathbf{v}_n + \mathbf{p}_{rn})$

Procedure

- I used MIT's Scratch, a visual programming language to make the triangle.
- My code can be seen on the right. It is a simple translation of the mathematical procedure (see previous slide).
- I did not backpack (=borrow) any code of others, and did not ask for help from parents or teachers for the programming.
- My program is available here (click green flag to run the program): https://scratch.mit.edu/projects/616965520/



Constant Conditions:

Independent Variable:
The vertices of the triangle. The chaos game should work with different kinds of triangles, not just equilateral ones.
I used four different conditions, which are variations of the triangle

* equilateral triangle
* obtuse triangle
* right triangle
* isosceles triangle

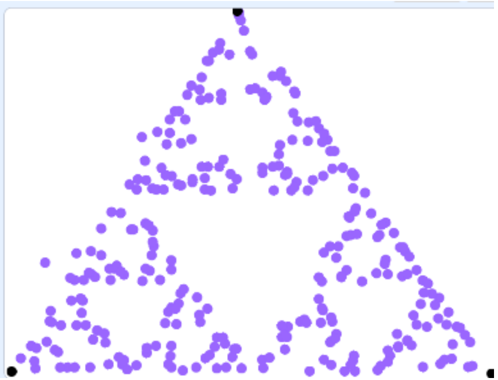
For each kind of triangle, the starting point appears at random. Sometimes it appears outside, sometimes inside of the vertices.

Dependent Variable
I examine whether a fractal pattern emerges in these different triangles and what it looks like.

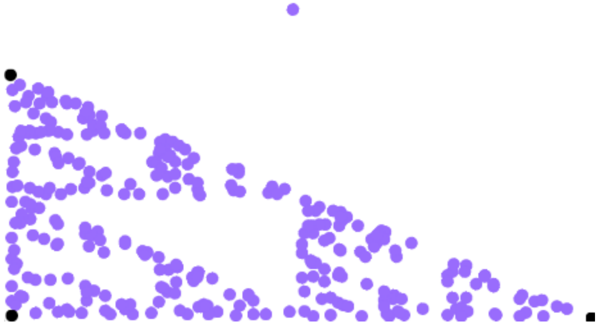
Constant Conditions
The rules of the chaos game.

Data and trials (1)

I ran the program five times for each of the kinds of triangle to see if the pattern keeps on emerging, so for a total of 20 trials. The results as you can see are visual.
Note that the maximum number of clones (dots drawn) in Scratch is 1000, so the fractal pattern never completely fills up. Here are pictures for each of the kinds of triangles.



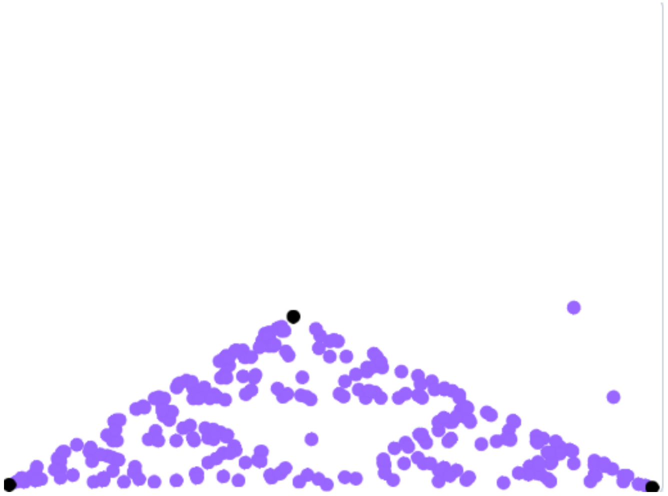
Equilateral triangle, trial 2: starting point was outside.



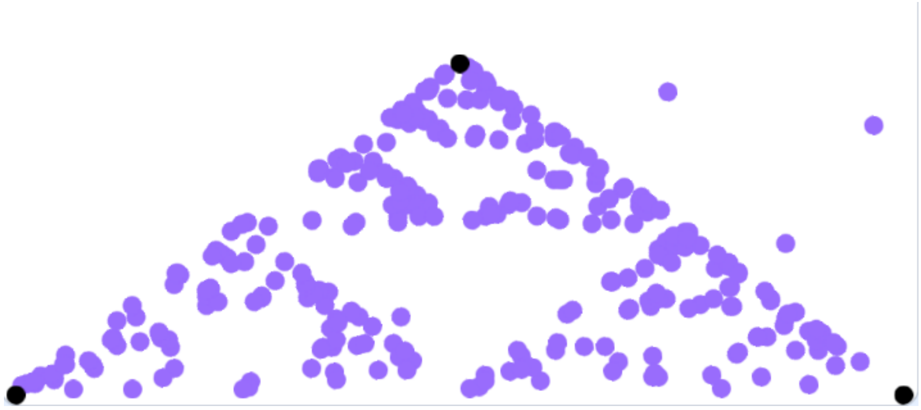
Right triangle, trial 5: the starting point is far outside, but yet the figure still appears.

Data and trials (2)

Here are more pictures of the triangles.
For each of the types of triangles, we see order emerge out of chaos.
The triangle shape is an *attractor* as we keep on getting it using the rules of the chaos game.



Obtuse triangle, trial 2: this looks a bit messier, but the shape does emerge.



Isosceles triangle, trial 1: quite a few points outside of the triangle, but the shape still comes in fractals.

Conclusion and Reflection:

I found out that when you play the chaos game with different kinds of triangles, a pattern emerges where the triangle's shape appears (often upside down) in a fractal pattern.

It was surprising to see that no matter where the starting point was, the fractal pattern reliably emerged.

The chaos game has real applications, for instance, it is used to make surfaces in computer games.

My experiment was limited by Scratch's limitation of only 1000 points. I hope to repeat it using a programming language such as Python, once I learn that.

As Benoit Mandelbrot (2010) said in his TED talk: "Bottomless wonders spring from simple rules repeated without end."

References

- Mandelbrot, Benoit (1982) *The Fractal Geometry of Nature.* WH Freeman and Co.
- Mandelbrot, Benoit (2010, shortly before he died). TED talk on Mandelbrot fractals and the art of roughness https://www.ted.com/talks/benoit_mandelbrot_fractals_and_the_art_of_roughness
- Lathrop, J. I., Lutz, J. H., & Summers, S. M. (2009). Strict self-assembly of discrete Sierpinski triangles. *Theoretical Computer Science*, *410*(4-5), 384-405.