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Date of Report: 9/29/01

The Period of a Pendulum

Purpose:

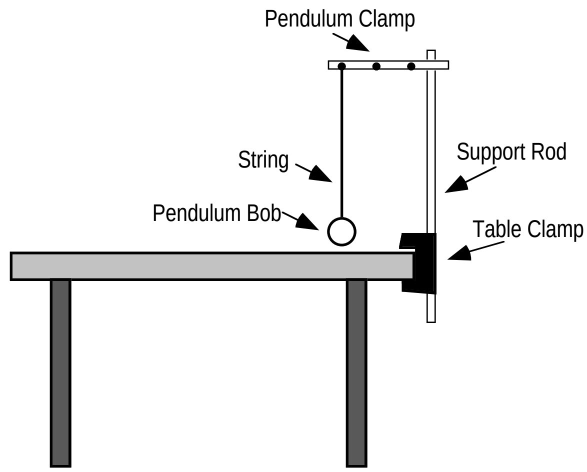
The purpose of this experiment is to investigate the effects of changing mass, amplitude, and length on the period of a pendulum.

Hypothesis:

a. If the mass of the pendulum bob is increased then the period will increase.
b. If the amplitude is increased then the period will increase
c. If the length of the pendulum is increased then the period will increase.

Equipment:

The apparatus for this experiment consists of a support stand, pendulum clamp, three strings and three objects (1 copper ball, 1 aluminum ball, and one large washer) to use as pendulum bobs. The equipment is set up as shown in the schematic diagram to the right. In addition a balance, a meter stick, and a stopwatch are required to collect the data required for this investigation.



Procedure:

This investigation involves three separate experiments which test factors which may affect the period of a pendulum. In each case the period is determined by timing the pendulum for 10 swings (back and forth) and then dividing this time by 10 to determine the time for a single swing.

Experiment A: The Effect of Changing Mass on the Period of a Pendulum
In this experiment the mass of the pendulum was changed by either completely changing the pendulum bob, or by combining bobs to get a mass different from individual bobs. We used the following combinations for our different bobs: a single aluminum sphere, a single copper sphere, a single galvanized washer, the aluminum sphere and the copper sphere, the copper sphere and the washer, and finally the aluminum sphere, the copper sphere and the washer. The mass of each of the three individual bobs was determined with a Harvard trip balance, and the masses of the combinations were determined by adding the masses of the individual components. The bob was released from a starting position 20 cm from the center point of the swing for each run. The length of the string was held constant at 70.5 cm. Four students timed each data run. Each of the four times was recorded. In this experiment mass was the independent variable and period was the dependent variable.

Experiment B: The Effect of Changing Amplitude on the Period of a Pendulum
For our purposes in this experiment, we defined the amplitude of swing for the pendulum as the distance from the point of release to the midpoint of the swing. The copper sphere/washer combination was used for each run so that the mass was held constant at 119.93 g. The length of the string was held constant at 70.5 cm. The sphere was pulled back from 5 cm to 30 cm in 5 cm increments, yielding six different amplitudes. Four students timed each data run. The table was marked with the center position and each of the six starting points. Each of the four times was recorded. In this experiment amplitude was the independent variable and period was the dependent variable.

Experiment C: Changing Length

For this experiment, we defined the length of the pendulum as the distance from the point of connection to the pendulum clamp to the point where the string was attached to the bob. The copper sphere/washer combination was used for each run so that the mass was held constant at 119.93 g. The amplitude held constant at 70.5 cm. The sphere was pulled back a distance of 20 cm for each run. The length of the string was changed starting at 70.5 cm and decreasing by 10 cm for each run until the string reached a length of 20.5 cm. Four students timed each data run. Each of the four times was recorded. In this experiment length was the independent variable and period was the dependent variable.

Raw Data:

MASS/PERIOD EXPERIMENT

length 70.5 cm amp. 20.0 cm

mass (g)	time for 10 swings (s)			
21.85	16.52	16.60	16.57	16.62
72.88	16.52	16.65	16.81	16.95
47.05	17.04	17.24	17.24	16.92
94.73	17.04	16.91	17.20	17.15
141.78	16.79	16.66	17.00	17.05
119.93	16.85	16.89	16.99	17.23

AMPLITUDE/PERIOD EXPERIMENT

mass 119.93g length 70.5 cm

amplitude (cm)	time for 10 swings (s)			
5.0	17.09	16.81	16.98	17.13
10.0	16.51	16.66	16.92	16.88
15.0	16.91	16.96	17.69	17.11
20.0	16.85	16.89	16.99	17.23
25.0	17.09	17.04	17.40	17.31
30.0	17.18	17.67	17.44	17.23

LENGTH/PERIOD EXPERIMENT

amplitude 20.0cm mass 119.93g

Length (cm)	Time for 10 swings (s)			
20.5	10.64	10.70	10.77	10.76
30.5	11.89	11.82	11.81	11.91
40.5	13.33	13.31	13.24	13.38
50.5	14.84	15.00	14.73	14.77
60.5	16.84	16.02	16.10	15.71
70.5	16.85	16.89	16.99	17.23

Evaluation of Data:

DATA FOR PERIOD VS. MASS EXPERIMENT						
mass (g)	**time for 10 swings (s)**				**average time (s)**	**period (s)**
21.85	16.52	16.60	16.57	16.62	16.58	1.66
47.05	17.04	17.24	17.24	16.92	17.11	1.71
72.88	16.52	16.65	16.81	16.95	16.73	1.67
94.73	17.04	16.91	17.20	17.15	17.08	1.71
119.93	16.85	16.89	16.99	17.23	16.99	1.70
141.78	16.79	16.66	17.00	17.05	16.88	1.69
Constants:	Length = 70.5 cm				Amplitude = 20.0 cm	

In the table above, the average time was calculated by adding the four time values in the data table and dividing by four. The period was determined by dividing the average time by 10 to obtain the time for a single swing.

Sample Calculations:

$$average\ time = \frac{t_1 + t_2 + t_3 + t_4}{4}$$

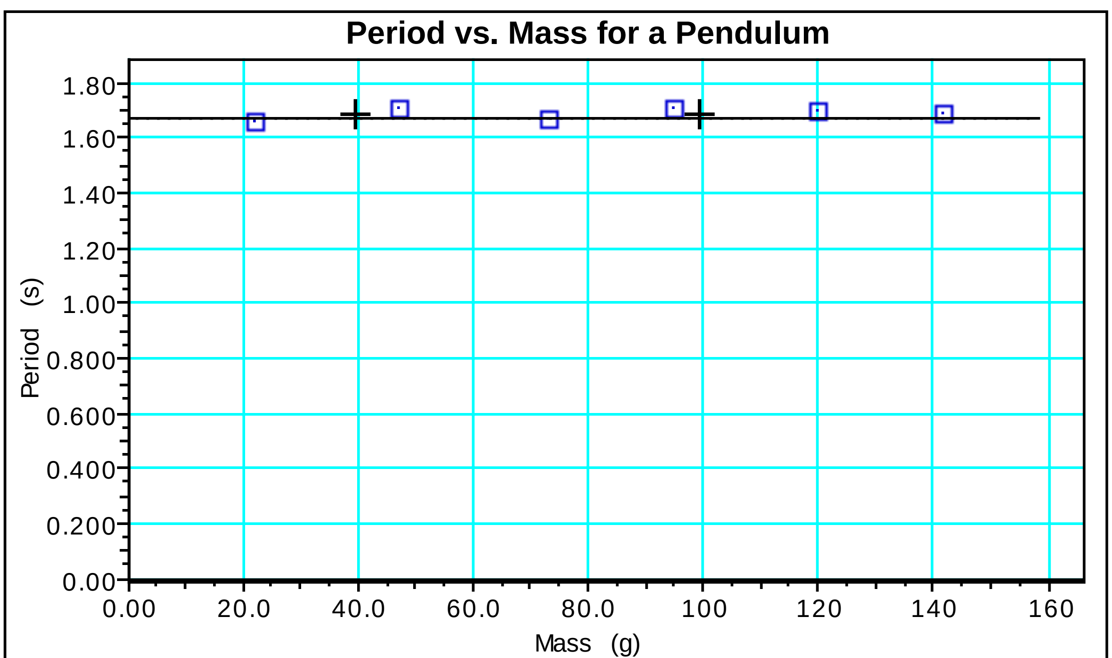
$$average\ time = \frac{16.52\,s + 16.60\,s + 16.57\,s + 16.62\,s}{4}$$

$$average\ time = 16.58\,s$$

$$period = \frac{average\ time}{10}$$

$$period = \frac{16.58\,s}{10}$$

$$period = 1.66\,s$$



Manual Curve Fit:

f(x)= A + B*x

A= 1.67 B= 0.00

Mean Square Error: 0.000767

Mathematical Analysis of Period vs. Mass Experiment:

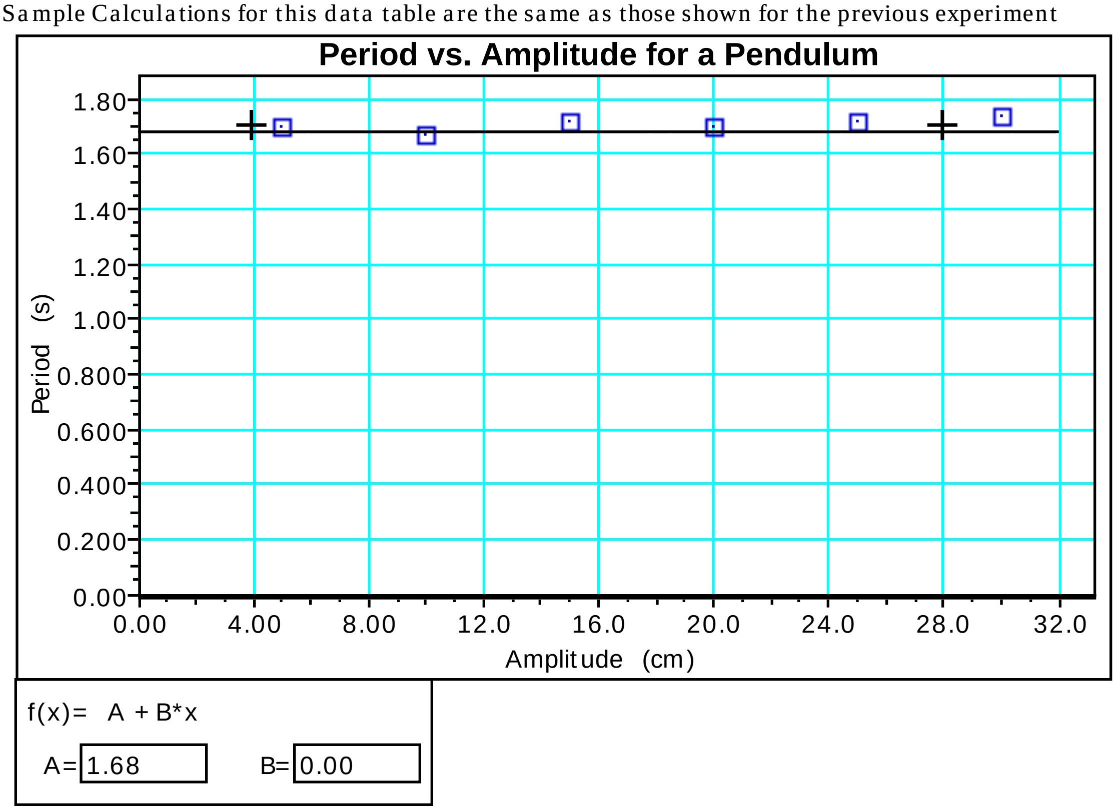
1. Period $\rightarrow$ T Mass $\rightarrow$ m
2. $y = mx + b$
3. $T = km + b$
4. $k = \dfrac{\Delta T}{\Delta m}$
5. $k = \dfrac{T_2 - T_1}{m_2 - m_1}$
6. $k = \dfrac{1.67\,s - 1.67\,s}{100.0\,g - 40.0\,g}$
7. $k = 0$
8. $b = 1.67\,s$
9. $T = 0 \cdot m + 1.56\,s$
10. $T = 1.67s$

The graph shows that there is no relationship between period and mass for a pendulum, since as the mass of the pendulum is increased, the period stays essentially constant. This is indicated by a slope which is essentially zero. The mathematical model which describes this relationship is T = 1.67 s, where T represents the period of the pendulum and 1.67 s is the “y-intercept” of the graph. This equation says that no matter what the mass of the pendulum is, the period stays constant at 1.67 s.

$$T = 1.67\ s$$

DATA FOR PERIOD VS. AMPLITUDE EXPERIMENT						
amplitude (cm)	time for 10 swings (s)				average time (s)	period (s)
5.0	17.09	16.81	16.98	17.13	17.00	1.70
10.0	16.51	16.66	16.92	16.88	16.74	1.67
15.0	16.91	16.96	17.69	17.11	17.17	1.72
20.0	16.85	16.89	16.99	17.23	16.99	1.70
25.0	17.09	17.04	17.40	17.31	17.21	1.72
30.0	17.18	17.67	17.44	17.23	17.38	1.74
Constants:	Length = 70.5 cm				Mass =119.93 g	

Sample Calculations for this data table are the same as those shown for the previous experiment



Mathematical Analysis of Period vs. Amplitude Experiment:

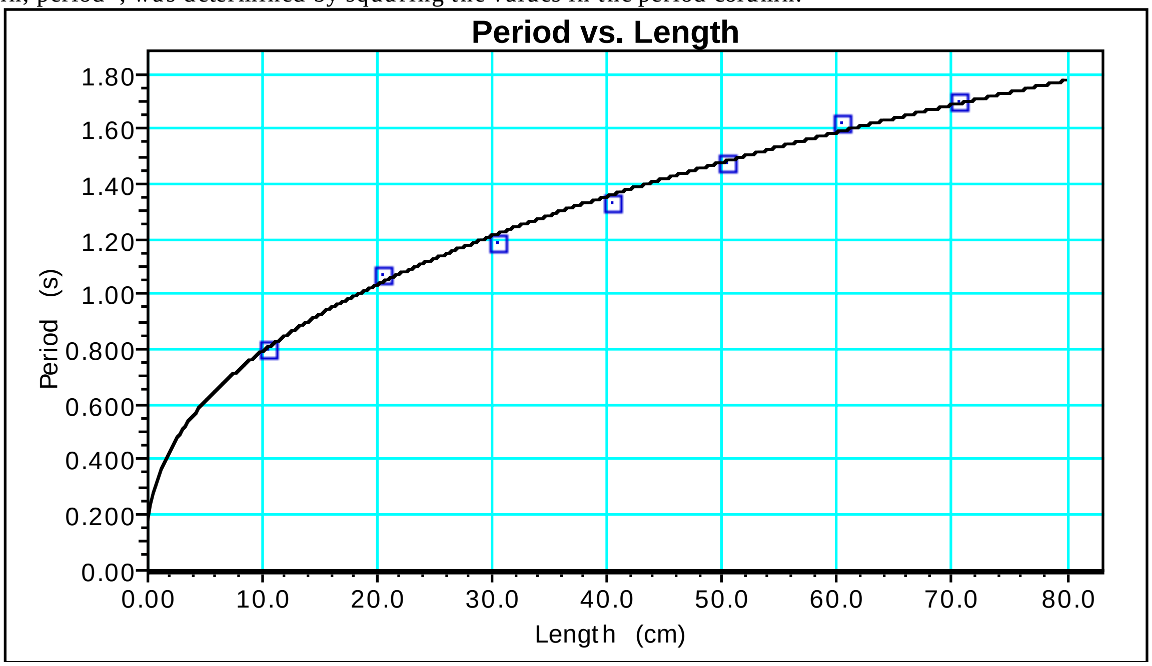
1. Period $\rightarrow$ T Amplitude $\rightarrow$ A
2. $y = mx + b$
3. $T = kA + b$
4. $k = \dfrac{\Delta T}{\Delta A}$
5. $k = \dfrac{T_2 - T_1}{A_2 - A_1}$
6. $k = \dfrac{1.68s - 1.68s}{28.0cm - 4.0g}$
7. $k = 0$
8. $b = 1.68s$
9. $T = 0 \cdot A + 1.68s$
10. $T = 1.68s$

This graph shows that there is no relationship between period and amplitude for a pendulum, since as the mass of the pendulum is increased, the period stays essentially constant. The slope of the Period vs. Amplitude graph is essentially zero. The mathematical model which describes this relationship is T = 1.68 s, where T represents the period of the pendulum and 1.68 s is the "y-intercept" of the graph. This equation says that no matter what the amplitude of the pendulum is, the period stays constant at 1.69 s.

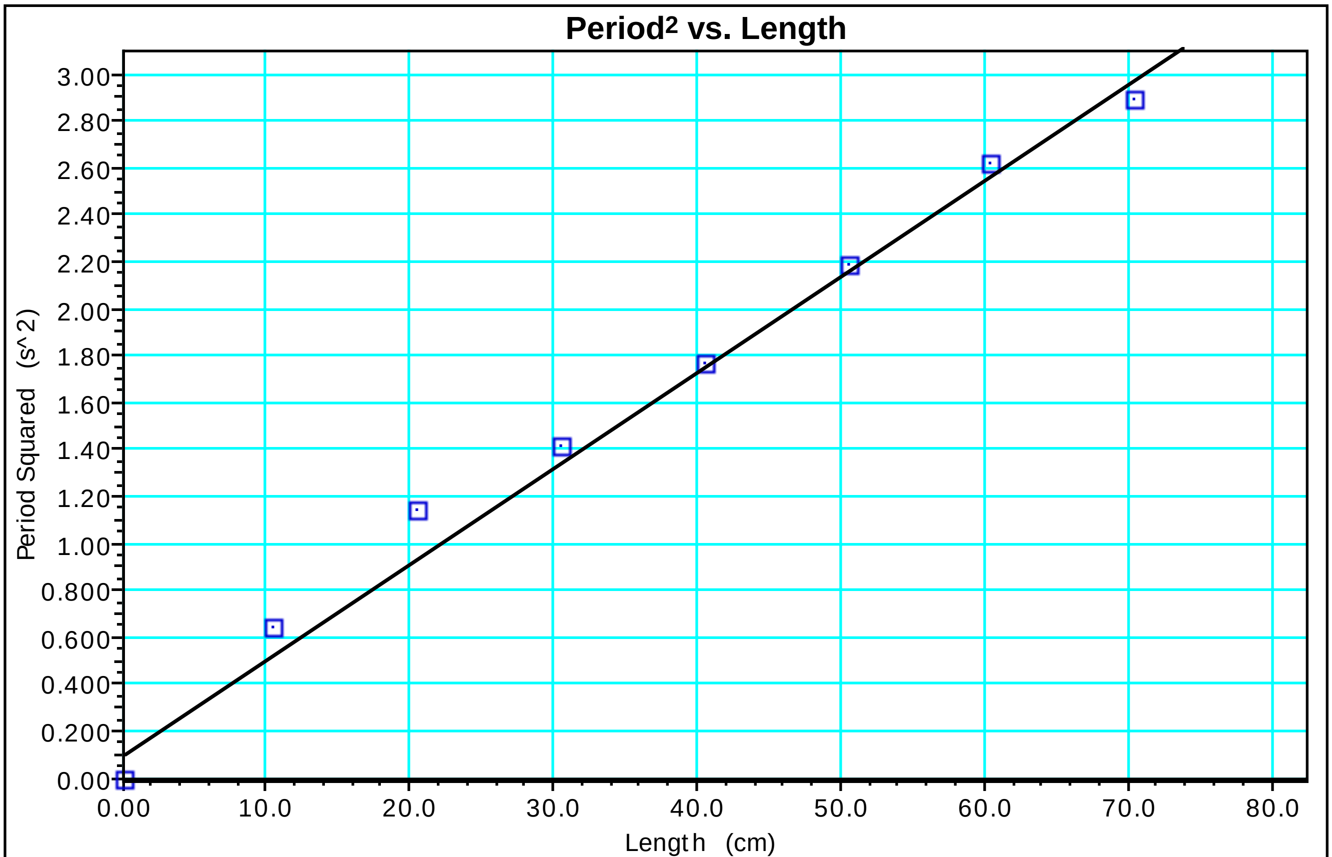
$$T = 1.69 \text{ s}$$

DATA FOR PERIOD VS. LENGTH EXPERIMENT							
length (cm)	**time for 10 swings (s)**				**average time (s)**	**period (s)**	**period^2 (s^2)**
10.5	7.92	8.01	7.97	8.02	7.98	0.80	0.64
20.5	10.64	10.70	10.77	10.76	10.72	1.07	1.15
30.5	11.89	11.82	11.81	11.92	11.86	1.19	1.41
40.5	13.33	13.31	13.24	13.38	13.32	1.33	1.77
50.5	14.84	15.00	14.73	14.77	14.83	1.48	2.20
60.5	16.84	16.02	16.10	15.71	16.17	1.62	2.61
70.5	16.85	16.89	16.99	17.23	16.99	1.70	2.89
Constants:	Mass = 119.93 g				Amplitude = 20.0 cm		

Sample Calculations for the average time and period are the same as those shown for the previous experiments. The last column, period^2, was determined by squaring the values in the period column.



This graph shows that as length increases, period increases but at a lower rate. It appears as if it could be a sideways opening parabola. I will test this by squaring Period and plotting a graph of Period^2 vs. Length.



Statistics:	Slope	Y Intercept	C.O.R.
Data Set 1	0.0399±0.00163	0.143±0.0689	0.995

Mathematical Analysis of Period vs. Length Experiment:

$$b = 0.143s^2$$

$$y_{\max} = 2.89s^2$$

$$\frac{b}{y_{\max}} = \frac{0.143s^2}{2.89s^2} = 0.049$$

Since the y - intercept is less than 5% of the maximum y - value, b can be reasonably treated as zero.

1. Period $\rightarrow$ T   Length $\rightarrow$ L
2. $T^2 \propto \mathrm{L}$
3. $T^2 = k\mathrm{L}$
4. $k = \dfrac{\Delta T^2}{\Delta L}$
5. $k = 0.399\dfrac{s^2}{cm}$ (slope calculated by computer)
6. $T^2 = 0.399\dfrac{s^2}{cm}\mathrm{L}$

This graph shows that there is a direct and linear relationship between length and period^2 for a pendulum. The "y-intercept" is essentially zero and can be ignored as shown by the 5% test. The mathematical model that results shows that:

$$T^2 = 0.399\ s^2/cm \cdot L$$

Error Analysis

The most significant source of error in this experiment deals with the reaction time of the experimenters. It is difficult to start the watch precisely at the beginning of a swing and equally difficult to stop the watch precisely at the end of a swing. The effect of this error is reduced by timing the pendulum for a number of swings (10 in this case). The total error for starting and stopping the watch is approximately the same regardless of the number of swings. The error associated with timing a single swing is therefore reduced by a factor of 10 by timing 10 swings and then dividing the time for 10 swings by 10 to yield the time for one swing.

Another possible source of error exists in the period vs. mass experiment. In order to perform a controlled experiment, we were supposed to keep the length of the string constant. Unfortunately, as mass was added to the pendulum string, the string stretched and made the string slightly longer. As shown in the third experiment, length does affect period, so small variations in period for the period vs. mass experiment are possibly attributable to the slight increase in length that accompanied the increases in mass.

This error along with the small error in measuring the mass of the pendulum are insignificant compared to the reaction time error that was a part of all three experiments.

There were no "accepted values" associated with this experiment, therefore a quantitative error analysis is not included in this section.

Conclusion

The purpose of this experiment was to determine what factors affect the period of a pendulum. In this investigation, the effects of changing mass, amplitude and length on the period of the pendulum were tested.

For the first experiment, changing the mass of the pendulum bob had no effect on the period of the pendulum. Period is <u>not</u> related to mass for a pendulum. This is shown by the horizontal line in the Period vs. mass graph. The slope of zero tells us that the period values stayed constant at 1.67 s as the mass was increased. The equation which describes this relationship is T = 1.67 s, where T represents the period of the pendulum and 1.67 s is the "y-intercept" of the graph. Since mass doesn't appear in the final equation, it tells us that regardless of the mass, the period stays constant at 1.67 s.

This result is inconsistent with my prediction. I predicted that increases in mass would result in increases in the period. I thought that increasing the mass would make it more difficult to speed up the pendulum, thus making it travel slower and take more time to complete a swing. Perhaps, however, increasing the mass increases the force pulling the pendulum toward the center, thus making it tend to travel faster and take less time. The lack of any effect of mass on the period may be because both of these are happening simultaneously, competing in opposite directions to change the period, and effectively "canceling" each other.

Conclusion, cont.

For the second experiment, changing the amplitude of the pendulum bob had no effect on the period of the pendulum. Period is not related to amplitude for a pendulum. The equation which describes this relationship is $T = 1.68$ s, where T represents the period of the pendulum and 1.68 s is the "y-intercept" of the graph. Again the slope of zero and the corresponding equation indicate that regardless of the amplitude, the period is constant at 1.68 s. This result is inconsistent with my prediction. I predicted that increases in amplitude would result in increases in the period. My prediction was based on the thought that increasing the distance the bob must travel would increase the time required to complete one swing.

This idea is reasonable if the bob traveled at a constant speed for the whole swing regardless of its distance from the center. It is possible that increasing the distance from the center also increased the force pulling the bob back toward the center, thus increasing the average speed that the bob traveled for each swing. This effect, coupled with the effect described by my prediction could account for the independence of amplitude and period.

For the third experiment, the graph shows that increasing the length of the pendulum increased the period, but not in a linear fashion. For equal increments of length, the increments of period were decreasing. Squaring period yielded a linear graph showing that period^2 is directly proportional to the length of the pendulum. This could also be rearranged to show that period it directly proportional to the square root of the pendulum. This graph can be described mathematically by the equation: $T^2 = 0.399\ s^2/cm \cdot L$, where T represents the period and L represents the length of the period.

I am not sure what the slope of this equation means. Its units are such that if the reciprocal of the slope were determined, acceleration units would result. As we designed this set of experiments, gravity was mentioned as a possible quantity that might affect period. Since a physical constant associated with gravity is the acceleration due to gravity, and it was held constant in this experiment, and the units of the slope are those of acceleration, it is possible that the slope is related to the acceleration due to gravity. Further experimentation would be required to support this idea, and appreciably changing the value of the acceleration due to gravity on the earth is a difficult thing to do. This result is consistent with my prediction, however my prediction was not specific enough to indicate the square root nature of the relationship between period and length.

In summary, our experiment and the graphical analysis which followed it indicate that the period of a pendulum is affected by its length, but not by its mass or amplitude. It would be interesting to try to devise an experiment that would allow us to investigate the effects that gravity might have on the period of the pendulum.