Singapore Math:

Using the bar model approach, Singapore textbooks enable students to solve difficult math problems—and learn how to think symbolically.

John Hoven and Barry Garelick

Here is a math problem you can solve easily:

> A man sold 230 balloons at a fun fair in the morning. He sold another 86 balloons in the evening. How many balloons did he sell in all?

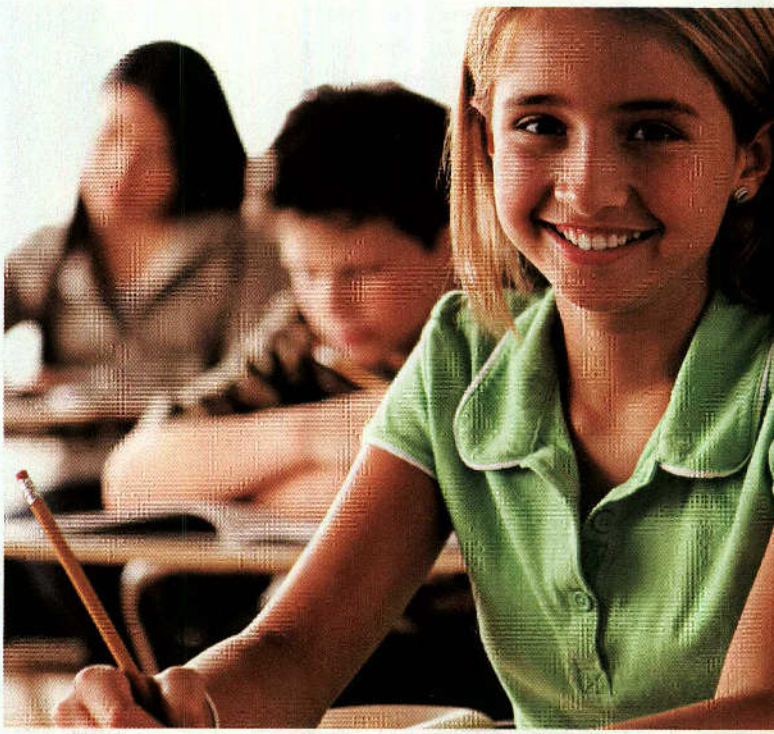
And here is one you can't:

> Lauren spent 20 percent of her money on a dress. She spent 2/5 of the remainder on a book. She had $72 left. How much money did she have at first?

In Singapore, where 4th and 8th grade students consistently come in first on international math exams, students learn how to solve both problems using the same *bar model* technique. Students first encounter the technique in 3rd grade, where they apply it to very simple problems like the first one. In grades 4 and 5, they apply the same versatile technique to more difficult, multistep problems. By grade 6, they are ready to solve really hard problems like the second one. With that solid foundation, students easily step into algebra. The bar modeling tool has taught them not only to solve math problems but also to represent them symbolically—the mainstay of algebraic reasoning.

Bar modeling is a specific variant of the common Draw a Picture mathematics problem-solving strategy. Because Singapore Math uses this one variant consistently, students know what kind of picture to draw. That's an advantage if the bar model is versatile enough to apply to many complex problems—and it is. It is especially useful for problems that involve comparisons, part-whole calculations, ratios, proportions, and rates of change. It communicates graphically and instantly the information that the learner already knows, and it shows the student how to use that information to solve the problem.

Singapore's textbooks are used in more than 600 schools in the United States and also by many homeschoolers. The books were discovered and drew high praise when mathematicians and teachers investigated why Singapore scored so high on international math exams. Homeschoolers and teachers like them for their simple and effective approach. Mathematicians like them for their logical structure, coherent curriculum, and focus on the skills necessary for success in algebra.



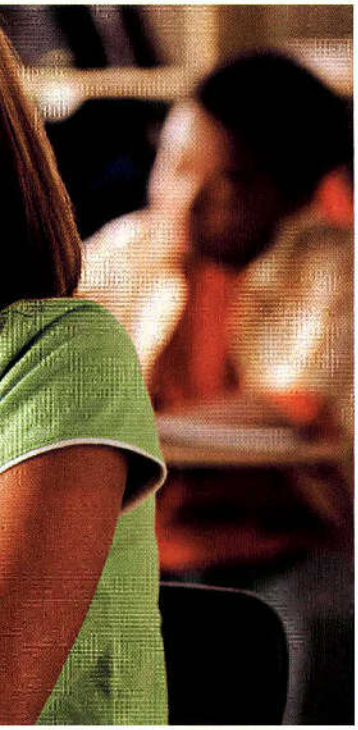
Scott Baldridge, a Louisiana State University mathematician, uses the Singapore Math texts in math courses for preservice teachers. He says,

> Students are treated by the curriculum as future adults who will need technical mathematics and the ability to do serious mathematical thinking in their careers.

Simple or Complex?

Deceptively Simple

Open a Singapore Math book to any page, and you may ask yourself, "How can a child *not* learn this?" Each concept is introduced with a simple explanation—often just a few words in a cartoon balloon. Students with weak reading and math skills benefit hugely from this direct simplicity.



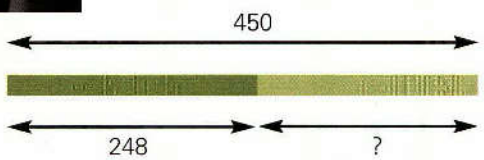
The first time 3rd graders see the bar model technique, they see how it's used in five demonstration problems.

The first two problems demonstrate the first basic variant of the technique—a single bar with two sections. This *part-whole model* works for simple addition and subtraction problems. (Part-whole relationships are a constant theme in Singapore Math, from 1st graders learning to add to 6th graders learning to divide fractions.) For example,

Daniel and Peter have 450 marbles.

Daniel has 248 marbles.

How many marbles does Peter have?[1]

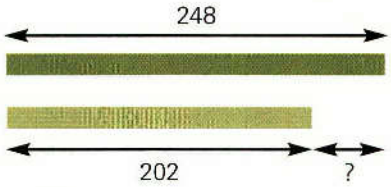


The next two problems demonstrate the second variant of the problem-solving technique—two bars to represent two different quantities. This comparison model works for problems that are solved by subtraction.

Daniel has 248 marbles.

Peter has 202 marbles.

Who has more marbles? How much more?

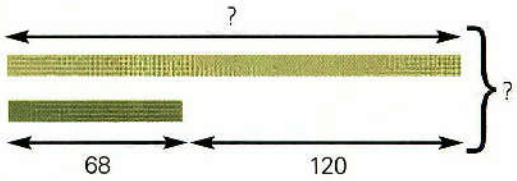


The final problem adds a side bracket joining the two bars. This variant works for two-step problems. That statement deserves emphasis: Singapore Math students are solving two-step problems in 3rd grade.

Mary had 120 more beads than Jill. Jill had 68 beads.

Step 1: How many beads did Mary have?

Step 2: How many beads did the two girls have altogether?

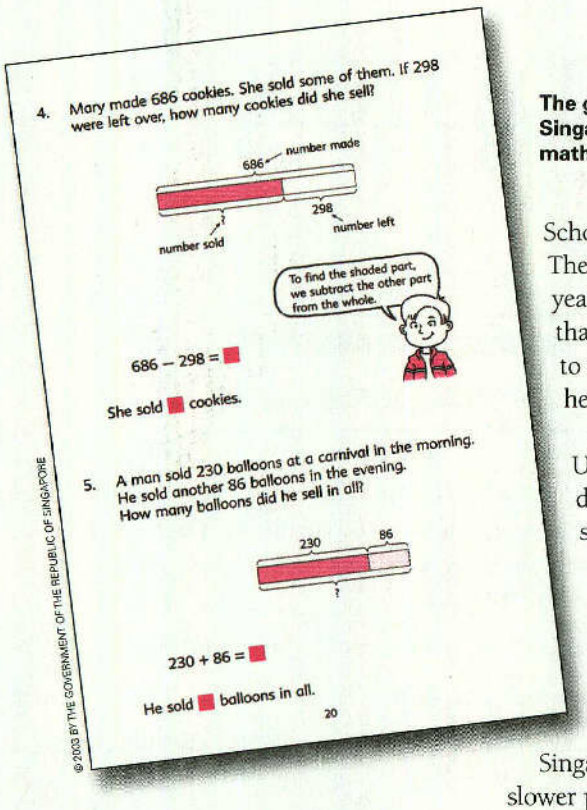


So the textbook's appearance of simplicity is deceptive. In this one lesson, Singapore Math students have already learned three basic variants of the bar model technique. They also apply these variants to problems that use a variety of synonyms for *add* and *subtract*. Discovering all the different ways to express the idea of math terms like *subtraction* is important for all students, but especially for English language learners struggling with word problems on a year-end assessment.

Although the explanations are simple and direct, they challenge students to think. When asked to find the difference between 9 and 6, for example, students understand that they need to use subtraction because of the part-whole relationship that has been used to teach them addition and subtraction. Because they know that 6 is part of 9, they easily understand how to find the difference.

Practice problems in Singapore Math are designed to teach skills step by step. The first few practice problems in a typical lesson might provide the appropriate bar model, whereas succeeding problems require the student to construct it.

The texts are carefully crafted so that students are presented



The grade 3 lesson shows how Singapore math textbooks present math concepts simply and directly.

with hints and examples for applying new concepts and algorithmic techniques, thus providing the scaffolding for learning. For example, the decimal division problem 0.6 ÷ 3 is made clear by showing six dimes (each equaling one 10th of a dollar) split into three equal groups. There are two dimes in each group, so six 10ths divided into three groups equals two 10ths: 0.6 ÷ 3 = 0.2. The lesson then asks students to extend this concept to more complex decimal division problems, such as 2 ÷ 4. Here students are given another hint—a cartoon character thinking "2 is 20 10ths." The students, having been led to discover how 10ths can be divided into groups, can now make another discovery and express whole numbers in terms of 10ths.

In-Depth Mastery

Singapore Math has been used for the past two years in grades 1–4 at the South River Primary School (grades K–2) and South River Elementary School (grades 3–5) in New Jersey. The school district, concerned about years of flat test scores in an area that is largely low income, decided to try Singapore's program after hearing about it at a workshop.

South River Principal Dorothy Unkel reports that teachers had difficulty at first making the transition to the new program:

> Singapore's approach is very teacher driven, much slower paced, and goes into much more depth. Teachers aren't used to that.

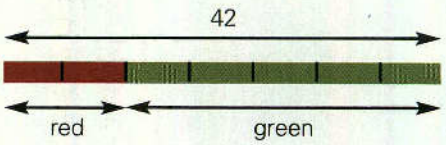
Singapore Math is able to teach at a slower pace and in more depth because it focuses instruction on the essential math skills recommended in the Curriculum Focal Points (National Council of Teachers of Mathematics, 2006). As a result, students make more rapid progress in those essential skills; for example, they learn multiplication in 1st grade. That surprising result—slower pace resulting in more rapid progress—works for students who perform on, above, or below grade level.

South River Assistant Superintendent Michael Pfister emphasizes that in the Singapore system, students achieve mastery, so schools do not need to reteach skills. That has implications for instructional grouping in a typical U.S. classroom, where students' math skills often range from two years below grade level to two years above. Singapore Math students should be grouped for instruction with the textbook that is at their level of understanding. Schools should use extra resources to help low-achieving students learn appropriate material at an accelerated pace, not to teach them material for which they have not mastered the necessary prerequisite skills.

Scaffolding the Way to Algebra

In Singapore Math, 3rd graders begin to apply the bar model technique to multiplication and division. By 4th grade, they are ready to apply it to fractions, as shown in the following problem:

> A grocer has 42 apples. 2/7 of them are red, and the rest are green. How many of them are green?



7 units = 42

1 unit = 42/7 = 6

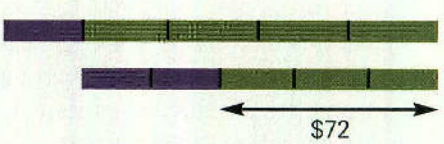
5 units = 6 x 5 = 30

There are 30 green apples.

Note that students are encouraged to see fractional "pieces" as units unto themselves. This will become important later when students encounter fractional division.

By 6th grade, students are solving complex, multistep problems like the one presented at the beginning of this article.

> Lauren spent 20 percent of her money on a dress. She spent 2/5 of the remainder on a book. She had $72 left. How much money did she have at first?



3 units = $72 (in the bottom bar model)

5 units = 5 x $72/3 = $120 = remainder

4 parts = $120 (in the top bar model)

5 parts = 5 x $120/4 = $150

By allowing students to identify the knowns and unknowns in a problem and their relation to one another, bar modeling sets the stage for the student to move to algebraic representation, as follows:

Amount Lauren had at first = x (length of the upper bar model)

After buying a dress, the remainder = r (length of the lower bar model)

From the lower bar model,

$72 = 3/5 of r = 3/5 x r

So 1/5 of r = $72/3 = $24, and

r = 5 x $24 = $120

From the upper bar model,

Amount spent on a dress = 0.20x = 1/5 of x

So R = 4/5 of X = 4/5 x x, which implies that

$120 = 4/5 of x = 4/5 x x

So 1/5 of x = $120/4 = $30, and

x = 5 x $30 = $150

For problems that are too complex to be represented pictorially through bar models, the compact conventions of algebraic symbolism—to which the student has been sequentially and methodically led—can come to the rescue.

Solving Problems, Reinforcing Concepts

It would be a mistake to think that the bar model approach to solving problems could be lifted out of Singapore Math and used by itself. Although bar modeling provides a powerful tool to represent and solve complex word problems, it also serves to explain and reinforce such concepts as addition and subtraction, multiplication and division, and fractions, decimals, percents, and ratios. If not linked to the concepts embedded in the lessons, the bar model would not necessarily be meaningful. The bar model and the basic skills embedded in the mathematical problems bootstrap each other.

Part-whole relationships are a constant theme in Singapore Math.

The end result of the Singapore Math program is that 6th graders can solve complex, multistep problems that most U.S. students, even those in algebra classes, would find challenging. According to a 2005 study by the American Institutes for Research (AIR), Singapore Math 6th grade problems are "more challenging than the released items on the U.S. grade 8 National Assessment of Education Progress" (p. xiii). AIR also found that

> the Singapore texts are rich with problem-based development in contrast to traditional U.S. texts that rarely get much beyond exposing students to the mechanics of mathematics and emphasizing the application of definitions and formulas to routine problems. (p. xii)

Singapore Math's trademark strategy—simple explanations for hard concepts—works! EL

[1] Problems and bar models are reproduced from *Singapore Primary Mathematics Teacher's Guides*, 3A (2003), 4A (2004), and 6B (2006), (U.S. ed.), Oregon City, OR: SingaporeMath.com. Copyright by SingaporeMath.com. Used with permission. Available: www.singaporemath.com

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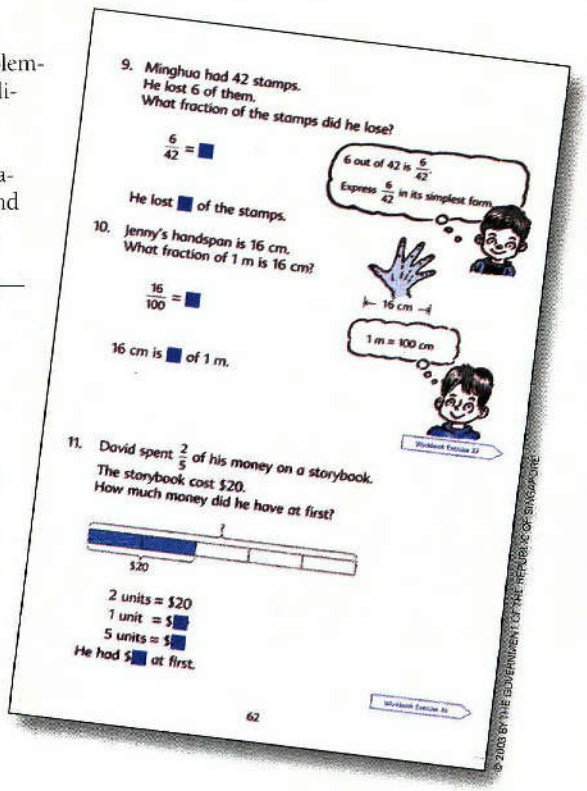

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