What is Calculus?

(Adapted from Elements of Calculus and Analytic Geometry, Addison-Wesley and Investigating Change: Putting Derivatives to Work, Curriculum Corporation)

Calculus is the mathematics of motion and change. Where there is motion or growth, where variable forces are at work producing acceleration, calculus is the right mathematics to apply. When things are changing, especially if they change in a smooth, gradual way, we may be able to understand the changes better. Calculus helps us to answer such questions as: How fast is this changing? When will it be increasing fastest? When will it start to decrease? When will it be biggest? When will it be smallest? Can we predict how big it will be and how fast it will be changing at some time in the future?

Calculus was first created to meet the mathematical needs of the scientists of the seventeenth century. Differential calculus dealt with the problem of calculating rates of change. It enabled people to define slopes of curves, to calculate the velocities and accelerations of moving bodies, to find the firing angle that gave a cannon its greatest range, and to predict the times when planets would be closest together or farthest apart. Integral calculus dealt with the problem of determining a function from information about its rate of change. It enabled people to calculate the future location of a body from its present position and a knowledge of the forces acting on it, to find areas of irregular regions in the plane, to measure the lengths of curves, and to locate the centers of mass of arbitrary solids.

Before the mathematical developments that culminated in the great discoveries of Sir Isaac Newton (1642-1727) and Baron Gottfried Wilhelm Leibniz (1646-1716), it took the astronomer Johannes Kepler (1571-1630) twenty years of concentration, record keeping, and arithmetic to discover the three laws of planetary motion that now bear his name:

1. Each planet travels in an ellipse that has one focus at the Sun.
2. The radius vector from the Sun sweeps out equal areas in equal intervals of time.
3. The squares of the periods of revolution of the planets about the sun are proportional to the cubes of their mean distances from the Sun. (If T is the length of a planet's year and D is the mean distance, then the ratio T^2/D^3 has the same constant value for all planets in the solar system.)

With calculus, deriving Kepler's laws from Newton's laws of motion can be done in one afternoon.

Today, calculus and its extensions in mathematical analysis are far reaching and the physicists, mathematicians, and astronomers who first invented the subject would be amazed to see what a profusion of problems it solves and what a wide range of fields now use it in the mathematical models that bring understanding about the universe and the world around us.

From an historical point of view, the calculus we use today is an accumulation of the contributions of many people. Its roots can be traced to classical Greek geometry, but its invention is chiefly the work of the scientists of the seventeenth century. Among these were Rene Decartes (1596-1650), Bonaventura Cavalieri (1598-1647), Pierre de Fermat (1601-1665), John Wallis (1616-1703), and James Gregory (1638-1675). The work culminated in the great creations of Newton and Leibniz.

The development of the calculus continued at a furious pace during the next century, and new applications to geometry, mechanics, engineering, and astronomy were found seemingly every day. Among the great contributors were several generations of Bernoullis, chiefly James Bernoulli (1654-1705) and his brother John Bernoulli (1667-1748) (the Bernoulli family was to mathematics what the Bach family was to music); Leonhard Euler (1707-1738) he was the key mathematical figure of the eighteenth century; Joseph Louis Lagrange (1736-1813); Adrien Marie Legendre (1752-1833); and many others.

The perfection of the logical structure behind the procedures of calculus was made by the mathematicians of the nineteenth century, among them Bernhard Bolzano (1781-1848), Augustin Louis Cauchy (1789-1857), and Karl Weierstrass (1815-1897). The nineteenth century also brought another round of spectacular extensions of calculus and great developments in mathematics beyond calculus.

Why is Calculus Useful?

Calculus is a mathematical tool with a very wide range of practical uses. Economists use calculus to forecast global trends. Oceanographers use calculus to formulate theories about ocean currents and meteorologists use it to describe the flow of air in the upper atmosphere. Biologists use calculus to forecast population size and to describe the way predators like foxes interact with their prey. Medical researchers use calculus to design ultrasound and x-ray equipment for scanning the internal organs of the body. Space scientists use calculus to design rockets and explore distant planets. Psychologists use calculus to understand optical illusions in visual perception. Physicists use calculus to design inertial navigation systems and to study the nature of time and the universe. Hydraulic engineers use calculus to find safe closure pattern valves in pipelines. Electrical engineers use it to design stroboscopic flash equipment and to solve the differential equations that describe current flow in computers. Sports equipment manufacturers use calculus to design tennis rackets and baseball bats. Stock market analysts use calculus to predict prices and assess interest rate risk. Physiologists use calculus to describe electrical impulses in neurons in the human nervous system. Drug companies use calculus to determine profitable inventory levels and timber companies use it to decide the most profitable time to harvest trees. The list is practically endless, for almost every professional field today uses calculus in some way.

More Specifically…

Optimization

Often it is important to find the cheapest or quickest way of doing something, or the way that uses the least amount of materials or makes the greatest profit. For example, we might want to design packaging which used the least amount of construction material for a given size, or which will hold the greatest quantity for a given amount of construction material. Questions like this, called *optimization problems*, arise very often in business, economics, architecture, agriculture and other areas. Calculus provides one of several mathematical techniques for solving such problems.

Economics
Economists use calculus to set up mathematical models which attempt to explain and predict changes in prices, unemployment, inflation and many other variables, while taking into account the effect of factors such as interest rates and taxes. These models are analyzed and the results may form the basis of recommendations to governments.

Biological, Medical and Social Sciences
Because calculus deals with change, it can be applied to any area of study where changes in one quantity bring about changes in another, as long as we can model the phenomena with functions which can be differentiated. It is used nowadays in many branches of the biological and medical sciences. There are not so many applications to the social sciences, but the statistical methods used widely in these disciplines depend on calculus for their theoretical development. Here are some examples in the biological and medical sciences.

The Spread of Infectious Diseases
When an epidemic of any infectious disease breaks out, health departments and hospitals need to be able to predict how fast it will spread, so that they can make plans to provide the necessary drugs, vaccines, hospital beds, or whatever else is necessary. Once again, calculus can be used to help make these predictions and to work out, for example, how many new cases per day we can expect to have at different stages in the progress of an epidemic.

The Concentration of Drugs in the Bloodstream
When you take a tablet or capsule or any form of medicine administered by mouth, it does not have an effect on you immediately. The drug passes to your stomach and intestines. From there it is gradually absorbed into your bloodstream, which carries it to all parts of your body. However, as the blood passes through your kidneys, the drug is removed along with other impurities. So the drug is absorbed into the body from the intestines at one rate, and removed from the body by the kidneys at another rate. Using calculus, we can set up mathematical models of drug concentrations in the form of equations involving rates of change. Solutions to these equations may help medical researchers work out what dose to give, and how often, to maintain the required concentration of the drug is a patient's body.

Population Growth
The population of the world keeps growing. How big will it be in 10, 20, 100, … years time? Will we be able to grow enough food to feed everyone? Will we be able to provide housing, education, transport, medical facilities and jobs for this increasing population? These are important questions for everyone in our society and they need to have discussed at all levels of government. In planning for the future, we need to estimate the size of the population at different times. Calculus can help in the creation of mathematical models of population growth which may enable us to predict future population size.

The Survival of Endangered Species
This is a different type of population problem. If an animal population is vulnerable to predators or hunters or introduced to diseases, it may not be able to reproduce fast enough to compensate for those killed, and the population will eventually die out. Again, calculus can help us to create a mathematical model incorporating birth rates and death rates and even the number of predators. Analyzing such a model can help us to understand how things are changing and to take the best course of action to prevent the species dying out.

Weather Forecasting
The weather is influenced by a great many different factors. Weather forecasters have to take account of temperature, atmospheric pressure, humidity and wind speed, all of which may be continually changing. To make accurate predictions, they need to gather information from many different places and at different heights above ground, but before they can use this information to make forecasts, they need to solve some very complicated equations. These are derived from mathematical models of how winds and temperature vary and they involve rates of change.

Physics and Engineering
Without calculus, most present day engineering would be impossible. Calculus provides mathematical models for motion which can be made extremely accurate. These help us to analyze and predict the motion of all kinds of objects, including space shuttles, satellites, planets and comets. Calculus has also played a part in the development of theories underlying the design of bridges, airplanes and dams, the generation and transmission of electric power, the flow of liquids in pipelines and the functioning of all kinds of machinery.

These days, in applications like these, calculus is more likely to form part of the theoretical development of the subject than to be used for on-the-job decision-making. Computers may be programmed to do all the necessary calculations and the person using them may not be aware that calculus is involved, but the ideas underlying the program will be based on derivatives and rates of change.

Assignment: Due Sunday, August 16th by 10:00pm

Email your teacher (katelynlong@claytonschools.net) with a response to the following questions.

1. **How has this reading changed or shaped your perception of calculus?**
2. **What did you find surprising?**
3. **What are you most excited to learn about?**

Be sure to include your name and class period.