HAT
Complex Numbers

9/26/17

Warm Up: Rewrite this function in vertex form. Sketch.

$f(x) = x^2 - 4x + 5$

AOS: $x = 2$

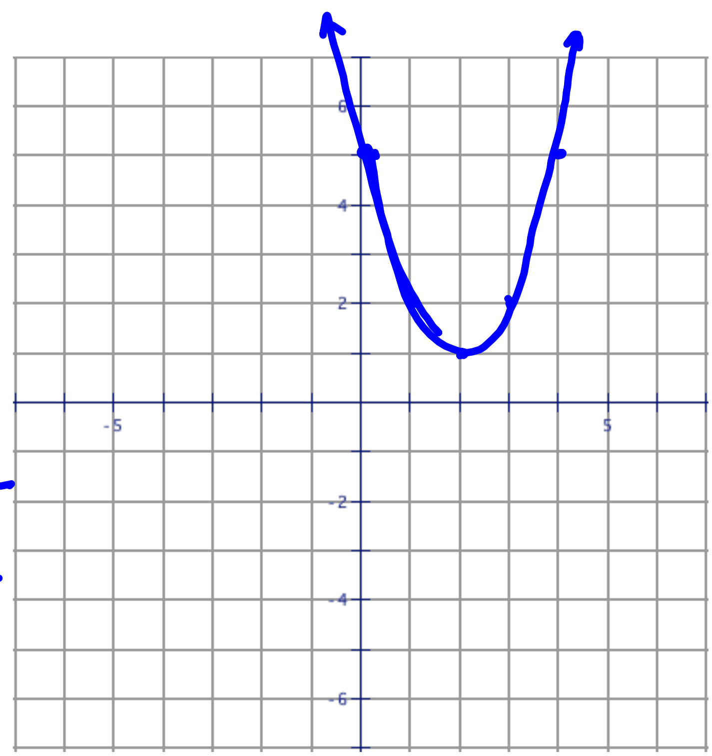
$f(2) = 1$ $(2,1)$

$f(x) = (x-2)^2 + 1$

$f(x) = x^2 - 4x + \underline{4} + 5 - \underline{4}$

$(x-2)(x-2)$

$f(x) = (x-2)^2 + 1$



Ex#1: Solve $x^2 - 4x + 5 = 0$

No real solutions

Q.F.

$$x = \frac{4}{2} \pm \frac{\sqrt{16-4(1)(5)}}{2}$$

Discriminant b^2-4ac

$$x = 2 \pm \frac{\sqrt{-4}}{2}$$

$$x = 2 \pm \frac{\sqrt{-1}\cdot\sqrt{4}}{\cancel{2}}$$

$$x = 2 \pm \sqrt{-1}$$

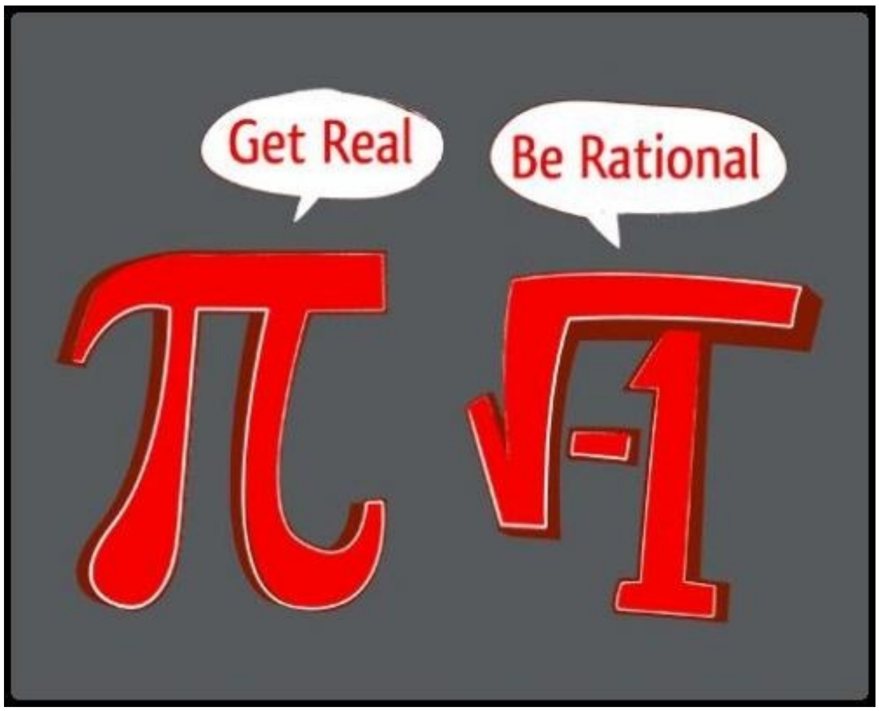
$$x = 2 \pm i$$

if $b^2-4ac>0$ 2 real sol.

if $b^2-4ac=0$ 1 real sol.

if $b^2-4ac<0$ no real sol. 2 complex sol.

Until now, we have just been stuck.
To bypass this problem,
we define the imaginary unit "i" with
$i = \sqrt{-1}$


Get Real
Be Rational

All of the numbers that we have worked with so far are classified as "real" (rational, irrational, integers, whole, natural).

Complex numbers have both a real part and an imaginary part.

They look like a+bi

$x = 2 + i$
$x = 2 - i$

where a is the real part and bi is the imaginary part.

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2$

$i^2 = -1$

Our focus today is on the arithmetic of imaginary and complex numbers.

Ex#2: Simplify $\sqrt{-9} \cdot \sqrt{-4}$ $= 3i \cdot 2i = 6i^2$

$3 \cdot \sqrt{-1}$ $= -6$

IMPORTANT: Numbers must be put in *i*-form BEFORE any operations are performed.

$i = i$

$i^2 = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = 1$

$i^5 = i^4 \cdot i = i$

$i^6 = i^4 \cdot i^2 = -1$

$i^7 = -i$

$i^8 = 1$

Ex#3: Simplify i^{53} and i^{2010}

$i^{48} = 1$

i^{53}

$(i^{52}) \cdot i$

i

$i^{2008} \cdot i^2$

$1 \cdot -1$

-1

i^{39}

$i^{36} \cdot i^3$

$1 \cdot -i = -i$

i^{217}

$i^{216} \cdot i^1$

$1 \cdot i = i$

Lucky for us, adding and subtracting complex numbers works just like you would expect it to...

Ex#4: Simplify each expression

$(2+3i)+(4-i)$ $= 6+2i$

$(-1+3i)-(2-i)$ $-3+4i$

$(1-\sqrt{-16})-(\sqrt{-9})$ $1-7i$

$1-4i-3i$

Multiplying also behaves as you would expect...

Ex#5: Simplify each expression

$-5(-1)=5$

$(3-5i)(2+i) = 6+3i-10i-5i^2$

$\boxed{11-7i}$

$4i(-1+2i)$

$-4i+8i^2$ $\boxed{-8-4i}$

$(4-3i)^2$ $(4-3i)(4-3i)$

Unfortunately, the procedure for dividing is less obvious...

Ex#6: Simplify $(2+i)\div(3-i)$

$$\frac{2+i}{3-i}\cdot\frac{3+i}{3+i}$$

Mult by conjugate of the demoninator

$$\frac{6+2i+3i+\overset{-1}{i^2}}{9+\cancel{3i}-\cancel{3i}-\underset{+1}{i^2}}$$

$$\frac{5+5i}{10}$$

$$\boxed{\frac{1}{2}+\frac{1}{2}i}$$

Assignment: pg. 250 #24, 26, 30, 33, 37, 39, 45, 54, 67, 69

September 26, 2017