HAT
Solving Trigonometric Equations

5/16/18

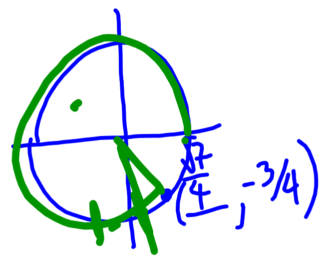
Warm Up:

Given $\sin\theta = -\frac{3}{4}$ where $\frac{3\pi}{2} \le \theta \le 2\pi$, find $\tan 2\theta$

$\frac{\sin(2\theta)}{\cos(2\theta)}$

$$\frac{2\sin\theta \cdot \cos\theta}{1 - 2\sin^2\theta}$$

$$\frac{2\left(-\frac{3}{4}\right)\cdot \cos\theta}{1-2\left(-\frac{3}{4}\right)^2} \qquad \cos\theta = \frac{\sqrt{7}}{4}$$



$$\cos^2\theta + \left(-\frac{3}{4}\right)^2 = 1$$

$$\cos^2\theta + \frac{9}{16} = 1$$

$$\sqrt{\cos^2\theta} = \sqrt{\frac{7}{16}}$$

$$\pm\frac{\sqrt{7}}{4}$$

$$\frac{\cancel{2}\left(-\frac{3}{\cancel{4}_2}\right)\left(\frac{\sqrt{7}}{4}\right)}{1-\cancel{2}\left(\frac{9}{\cancel{16}_8}\right)} = \frac{-\frac{3\sqrt{7}}{8}}{-\frac{1}{8}} \qquad 3\sqrt{7}$$

Solve i. on the interval $[0,2\pi)$ $\cancel{[0,2\pi]}$

ii. for ALL solutions.

EX1: $\cos x + \sqrt{2} = -\cos x$

$\sqrt{2} = -2\cos x$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \cos^{-1}(\cos x)$

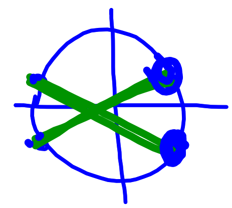
$\frac{3\pi}{4}+2\pi n \left\{ \frac{5\pi}{4}+2\pi n \right. = x$

i) $x = 3\pi/4$ OR $x = 5\pi/4$

ii) $x = 3\pi/4 + 2\pi n$ OR $x = 5\pi/4 + 2\pi n$

EX2: $2\sin^2 x = \frac{1}{2}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{4}}$



$\sin^{-1}(\sin x) = \sin^{-1}(\pm 1/2)$

i) $x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

ii) $x = \pi/6 + \pi n$

$x = 5\pi/6 + \pi n$

$\pm \pi/6 + \pi n$

Strategies for Solving:

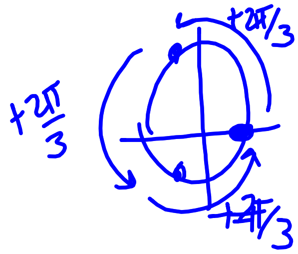
STRATEGY #1: Factoring

EX3: Solve $2\cos^2 x - \cos x - 1 = 0$

$w = \cos x$

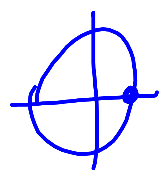
$$2w^2 - w - 1 = 0$$

$$(2w+1)(w-1) = 0$$



$$w = -\tfrac{1}{2} \quad w = 1$$

$$\cos x = -\tfrac{1}{2} \quad \cos x = 1$$



i) $x = 0, \frac{2\pi}{3}, 4\pi/3$

ii) $x = 0 + \frac{2\pi}{3}n$

$$x = \frac{2\pi}{3}n$$

STRATEGY #2: Using IDENTITIES

EX4: Solve

$$2\sin^2 x + 3\cos x - 3 = 0$$

$$2(1-\cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$-2\cos^2 x + 3\cos x - 1 = 0$$

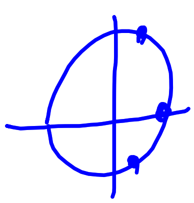
$$2\cos^2 x - 3\cos x + 1 = 0 \qquad w = \cos x$$

$$2w^2 - 3w + 1 = 0$$

$$(2w-1)(w-1) = 0$$

$$w = \tfrac{1}{2} \quad w = 1$$

$$\cos x = \tfrac{1}{2} \quad \cos x = 1$$



i) $x = 0, \pi/3, 5\pi/3$

ii) $x = 2\pi n$

$x = \pi/3 + 2\pi n$

$x = 5\pi/3 + 2\pi n$

EX5: Solve

$$3\sec^2 x - 2\tan^2 x - 4 = 0$$

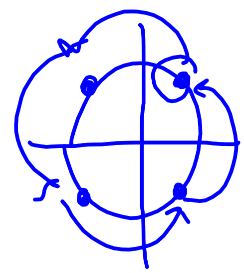
$$3(\tan^2 x + 1) - 2\tan^2 x - 4 = 0$$

$$3\tan^2 x + 3 - 2\tan^2 x - 4 = 0$$

$$\tan^2 x - 1 = 0$$

$$\sqrt{\tan^2 x} = \sqrt{1}$$

$$\tan x = \pm 1$$



i) $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

ii) $x = \pi/4 + \pi/2\, n$

$$3\sec^2 x - 2(\sec^2 x - 1) - 4 = 0$$

$$3\sec^2 x - 2\sec^2 x + 2 - 4 = 0$$

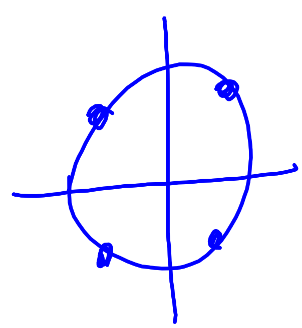
$$\sec^2 x - 2 = 0$$

$$\sqrt{\sec^2 x} = \sqrt{2}$$

$$\sec x = \pm\sqrt{2}$$

$$\cos x = \pm\frac{1}{\sqrt{2}}$$

$$\cos x = \pm\frac{\sqrt{2}}{2}$$



STRATEGY #3: Substitution

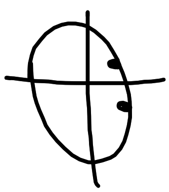
EX1: Solve $2\cos 2x - \sqrt{3} = 0$

$$2\cos 2x = \sqrt{3}$$

$$\cos 2x = \frac{\sqrt{3}}{2}$$

$w = 2x$

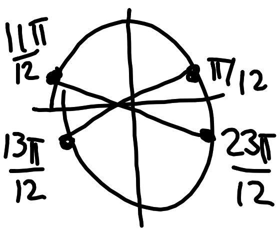
$$\cos w = \frac{\sqrt{3}}{2}$$



$$w = \frac{\pi}{6} + 2\pi n \quad \text{OR} \quad w = \frac{11\pi}{6} + 2\pi n$$

$$\frac{2x}{2} = \frac{\frac{\pi}{6} + 2\pi n}{2} \quad \text{OR} \quad \frac{2x}{2} = \frac{\frac{11\pi}{6} + 2\pi n}{2}$$

ii) $\boxed{x = \pi/12 + \pi n \quad \text{OR} \quad x = \frac{11\pi}{12} + \pi n}$



i) $\boxed{x = \pi/12,\ 11\pi/12,\ 13\pi/12,\ 23\pi/12}$

Assignment:

page 905 #30, 32, 33, 37, 42, 45, 49, 50, 51, 54, 59

Warm Up: Find ALL solutions.

$$\cos 2\theta + \cos\theta = -1$$

EX2: Solve $\sin^2 3x - 2\sin 3x + 1 = 0$

Assignment: WS Solving Trig Equations

<u>For Homework Points:</u>

Show me Identity Proofs by tomorrow.

This worksheet by Friday.