What if α and β are equal?

$\sin(\alpha+\alpha) =$ $\sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$2\sin\alpha\cos\alpha$

$\cos(\alpha+\alpha) =$ $\cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\cos^2\alpha - \sin^2\alpha$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \cot^2\theta = \csc^2\theta$ $\quad \tan^2\theta + 1 = \sec^2\theta$

Sum & Difference Identities:

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

Double Angle:

$\sin(2\theta) = 2\sin\theta \cdot \cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\quad \cos^2\theta - (1 - \cos^2\theta)$

$\cos(2\theta) = 1 - 2\sin^2\theta$ $\quad 2\cos^2\theta - 1$

$\cos(2\theta) = 2\cos^2\theta - 1$

Ex6: Prove $\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$

$\cancel{\cos\alpha\cdot\cos\beta} - \sin\alpha\sin\beta - (\cancel{\cos\alpha\cdot\cos\beta} + \sin\alpha\cdot\sin\beta)$

$-2\sin\alpha\cdot\sin\beta$ ✓

Ex7: Prove $\cos\left(x-\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{3}\right)=\cos x$

$\cos x \cdot \cos\frac{\pi}{3} + \cancel{\sin x \cdot \sin\frac{\pi}{3}} + \cos x \cos \pi/3 - \cancel{\sin x \cdot \sin \pi/3}$

$2\cos x \cdot \cos \pi/3$

$2 \cdot 1/2 \cdot \cos x$

$\cos x$ ✓

Ex8: Prove $\frac{\tan\theta}{\tan\theta}\cdot\frac{\tan\theta}{1+\sec\theta}+\frac{1+\sec\theta}{\tan\theta}\cdot\frac{1+\sec\theta}{1+\sec\theta}=2\csc\theta$

$$\frac{\tan^2\theta}{\tan\theta\cdot(1+\sec\theta)}+\frac{(1+\sec\theta)^2}{\tan(1+\sec\theta)}$$

$$\frac{\tan^2\theta+(1+\sec\theta)^2}{\tan(1+\sec\theta)}$$

$$\sec^2\theta \quad \frac{\tan^2\theta+1+2\sec\theta+\sec^2\theta}{\tan(1+\sec\theta)}$$

$$\frac{2\sec^2\theta+2\sec\theta}{\tan\theta(1+\sec\theta)}$$

$$\frac{2\sec\theta\cancel{(\sec\theta+1)}}{\tan\theta\cancel{(1+\sec\theta)}}$$

$$2\cdot\frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}$$

$$\frac{2}{\sin\theta}=2\csc\theta \checkmark$$