HAT
Sum and Difference Identities

5/10/18

WarmUp: Use a counterexample to show

$$\cos(\alpha+\beta) \neq \cos\alpha + \cos\beta$$

$\alpha = \frac{\pi}{2}$ $\quad \cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right) \neq \cos\frac{\pi}{2} + \cos\frac{\pi}{4}$

$\beta = \pi/4$ $\quad \cos\left(\frac{3\pi}{4}\right) \neq 0 + \frac{\sqrt{2}}{2}$

$-\frac{\sqrt{2}}{2} \neq \frac{\sqrt{2}}{2}$

$\alpha = \frac{\pi}{3}$

$\beta = \frac{2\pi}{3}$ $\quad \cos\left(\frac{\pi}{3}+\frac{2\pi}{3}\right) \neq \cos(\pi/3) + \cos(2\pi/3)$

$-1 \neq \frac{1}{2} - \frac{1}{2}$

$-1 \neq 0$

Sum and Difference Identities

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

OR:

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$
$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

Ex1: Find the EXACT VALUE.

a) $\cos\left(\frac{5\pi}{12}\right)$

$\cos\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$

$\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$\cos(\pi/6)\cdot\cos(\pi/4)-\sin(\pi/6)\cdot\sin(\pi/4)$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

b) $\sin 285^\circ$

$\sin(225+60)$

$\sin 225\cdot\cos 60+\cos 225^\circ\sin 60^\circ$

$-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}+-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$-\frac{\sqrt{2}}{4}+-\frac{\sqrt{6}}{4}$

$\frac{-\sqrt{2}-\sqrt{6}}{4}$

Ex2: Find the exact value.

$$\overset{\alpha}{\cos\frac{3\pi}{8}}\ \overset{\beta}{\cos\frac{5\pi}{8}} + \overset{\alpha}{\sin\frac{3\pi}{8}}\ \overset{\beta}{\sin\frac{5\pi}{8}}$$

$\alpha - \beta$

$\cos\left(\frac{3\pi}{8} - \frac{5\pi}{8}\right)$	$\cos\left(\frac{5\pi}{8} - \frac{3\pi}{8}\right)$
$\cos\left(-\frac{2\pi}{8}\right)$	$\cos\left(\frac{2\pi}{8}\right)$
$\cos(-\pi/4)$	$\cos(\pi/4)$
$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

Ex3: Prove $\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$

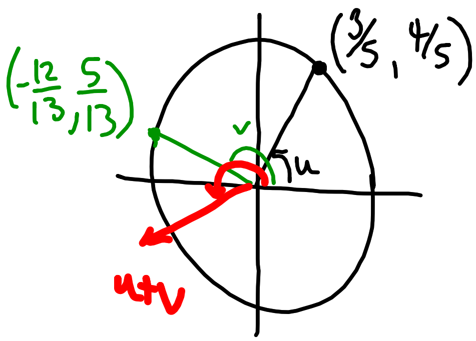
$\cos\frac{\pi}{2} \cdot \cos\theta + \sin\frac{\pi}{2} \cdot \sin\theta$

$\cancel{0 \cos\theta} + 1 \cdot \sin\theta$

$\sin\theta$

Ex4: Given $\sin u = \frac{4}{5}$, $0 < u < \frac{\pi}{2}$, $\cos v = -\frac{12}{13}$,

and $\frac{\pi}{2} < v < \pi$, find the exact value of $\sin(u+v)$



$\cos^2 u + \left(\frac{4}{5}\right)^2 = 1$

$\cos^2 u + \frac{16}{25} = 1$

$\sqrt{\cos^2 u} = \pm\sqrt{\frac{9}{25}}$

$\cos u = \pm\frac{3}{5}$

$\left(-\frac{12}{13}\right)^2 + \sin^2 v = 1$

$\frac{144}{169} + \sin^2 v = 1$

$\sqrt{\sin^2 v} = \sqrt{\frac{25}{169}}$

$\sin v = \pm\frac{5}{13}$

$\sin(u+v) = \sin u \cdot \cos v + \cos u \cdot \sin v$

$= \frac{4}{5} \cdot \frac{-12}{13} + \frac{3}{5} \cdot \frac{5}{13}$

$= \frac{-48}{65} + \frac{15}{65}$

$= \frac{-33}{65}$

EX5: Givesin $\theta = \frac{1}{3}$ an$0 < \theta < \frac{\pi}{2}$, find

a) $\sin 2\theta$

$\sin(\theta + \theta)$

$\sin\theta \cdot \cos\theta + \cos\theta \sin\theta$

$2 \cdot \sin\theta \cdot \cos\theta$

$2\left(\frac{1}{3}\right) \cdot \left(\frac{\sqrt{8}}{3}\right)$

$\frac{2\sqrt{8}}{9}$ or $\frac{4\sqrt{2}}{9}$

b) $\cos 2\theta$

$\cos(\theta + \theta)$

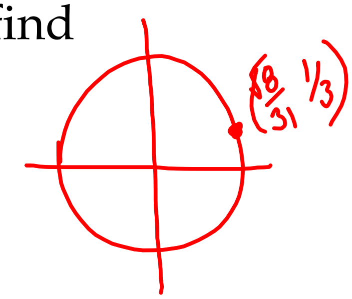
$\cos\theta \cdot \cos\theta - \sin\theta \cdot \sin\theta$

$\cos^2\theta - \sin^2\theta$

$\left(\frac{\sqrt{8}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$\frac{8}{9} - \frac{1}{9}$

$7/9$



$(1/3)^2 + \cos^2\theta = 1$

$\frac{1}{9} + \cos^2\theta = 1$

$\sqrt{\cos^2\theta} = \sqrt{\frac{8}{9}}$

$\cos\theta = \frac{\pm\sqrt{8}}{3}$

What if α and β are equal?

$$\sin(\alpha+\alpha)=$$

$$\cos(\alpha+\alpha)=$$

Ex6: Prove $\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$

Ex7: Prove $\cos\left(x-\frac{\pi}{3}\right)+\cos\left(x+\frac{\pi}{3}\right)=\cos x$

Ex8: Prove $$\frac{\tan\theta}{1+\sec\theta}+\frac{1+\sec\theta}{\tan\theta}=2\csc\theta$$

Assignment:

page 889 #14, 15, 19, 25, 28, 35, 38

page 897 #13*, 14*, 31*, 34*

*Only find $\sin 2\theta$ and $\cos 2\theta$

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