HAT
Recursive to Explicit,
Explicit to Recursive!

3/8/18

For each sequence:

Write the first 4 terms.

If the formula is explicit, write the recursive formula.

If the formula is recursive, write the explicit formula.

Prove that the formulas are equivalent by starting with the explicit formula.

EX #1: $\begin{cases} t_1 = -23 \\ t_{n+1} = t_n + 8 \end{cases}$ ← recursive

$-23, -15, -7, 1, 9, \ldots$

$t_n = -23 + 8(n-1)$

$t_n = 8n - 31$ ← Explicit

PROOF: 1) Show that both formulas give the same 1st term.

$t_n = 8n - 31$

$t_1 = 8(1) - 31$

$t_1 = -23$ ✓

2) Show the explicit gives the same reccurance relation as the recursive formula.

$t_n = 8n - 31$

$t_{n+1} = 8(n+1) - 31$

$t_{n+1} = 8n + 8 - 31$

$t_{n+1} = t_n + 8$ ✓

EX #2: $t_n = -4n + 7$ ← Explicit

$3, -1, -5, -9, -13, -17, \ldots$

$$\begin{cases} t_1 = 3 \\ t_{n+1} = t_n - 4 \end{cases}$$

$a_n = a_1 + d(n-1)$

$3 + -4(n-1)$

$3 - 4n + 4$

$7 - 4n$

PROOF: 1) $t_n = -4n + 7$

$t_1 = -4(1) + 7$

$t_1 = 3$ ✓

2) $t_n = -4n + 7$

$t_{n+1} = -4(n+1) + 7$

$t_{n+1} = -4n - 4 + 7$

$t_{n+1} = t_n - 4$ ✓

EX #3: $\begin{cases} t_1 = 3 \\ t_{n+1} = t_n \cdot 4 \end{cases}$ Recursive

$3, 12, 48, 192, \ldots$

Explicit: $t_n = 3(4)^{n-1}$

PROOF: 1) $t_n = 3(4)^{n-1}$

$t_1 = 3(4)^{1-1}$

$t_1 = 3$ ✓

2) $t_n = 3(4)^{n-1}$

$t_{n+1} = 3(4)^{n+1-1}$

$t_{n+1} = 3(4)^{n-1+1}$

$t_{n+1} = 3(4)^{n-1} \cdot 4^1$

$t_{n+1} = t_n \cdot 4$ ✓

EX #4: $a_n = 2n^2 + 1$

$3, 9, 19, 33, 51, \ldots$

$+6 \quad +10 \quad +14 \quad +18$

$+4 \quad +4 \quad +4$

$4n+2$

$6+4(n-1)$

$$\begin{cases} a_1 = 3 \\ a_{n+1} = a_n + 4n + 2 \end{cases}$$

1) $a_n = 2n^2 + 1$

$a_1 = 2(1)^2 + 1$

$a_1 = 3$ ✓

2) $a_n = 2n^2 + 1$

$a_{n+1} = 2(n+1)^2 + 1$

$a_{n+1} = 2(n^2 + 2n + 1) + 1$

$a_{n+1} = 2n^2 + 4n + 2 + 1$

$a_{n+1} = a_n + 4n + 2$ ✓