

HAT
Recursive to Explicit,
Explicit to Recursive!

3/8/18

For each sequence:

Write the first 4 terms.

If the formula is explicit, write the recursive formula.

If the formula is recursive, write the explicit formula.

Prove that the formulas are equivalent by starting with the explicit formula.

EX #1: $\begin{cases} t_1 = -23 \checkmark \\ t_{n+1} = t_n + 8 \end{cases}$ Recursive
 Arithmetic
 $-23, -15, -7, 1, 9, \dots$

$$t_n = -23 + 8(n-1)$$

$$t_n = -31 + 8n$$

PROOF: 1) $t_1 = -31 + 8(1)$
 $t_1 = -23 \checkmark$

2) $t_n = -31 + 8n$
 $t_{n+1} = -31 + 8(n+1)$
 $t_{n+1} = -31 + 8n + 8$
 $t_{n+1} = t_n + 8 \checkmark$

EX #2:

$$t_n = -4n + 7 \leftarrow \text{Arithmetic}$$

EXPLICIT

$$3, -1, -5, -9, -13, \dots$$

$$\text{Recursive: } \begin{cases} t_1 = 3 \\ t_{n+1} = t_n - 4 \end{cases}$$

Proof: $\begin{matrix} \nearrow \\ \text{Start} \\ \text{w/ explicit} \end{matrix}$

- 1) $t_n = -4n + 7$
 $t_1 = -4(1) + 7$
 $t_1 = 3 \checkmark$
- 2) $t_n = -4n + 7$
 $t_{n+1} = -4(n+1) + 7$
 $t_{n+1} = -4n - 4 + 7$
 $t_{n+1} = t_n - 4 \checkmark$

EX #3: $\begin{cases} t_1 = 3 \\ t_{n+1} = t_n \cdot 4 \end{cases}$ *Geometric*

3, 12, 48, 192, 768, ...

Explicit:
 $t_n = 3(4)^{n-1}$

Proof: 1) $t_n = 3(4)^{n-1}$
 $t_1 = 3(4)^{1-1}$
 $t_1 = 3 \checkmark$

2) $t_n = 3(4)^{n-1}$
 $t_{n+1} = 3(4)^{n+1-1}$
 $t_{n+1} = 3(4)^{n-1+1}$
 $t_{n+1} = 3(4)^{n-1} \cdot 4^1$
 $t_{n+1} = t_n \cdot 4 \checkmark$

EX #4: $a_n = 2n^2 + 1$

