HAT
Recursive Definitions

3/5/18

From yesterday...

Write $.\overline{27}$ as a fraction.

Warm Up: Evaluate $\sum_{n=1}^{\infty} 6 \cdot \left(\frac{1}{4}\right)^{n-1}$

Does this series converge or diverge?

$6 + \frac{3}{2} + \frac{6}{16} + \ldots\ldots$

$S_\infty = \frac{a_1}{1-r}$

$S_\infty = \frac{6}{1 - 1/4}$

$S_\infty = \frac{6}{3/4}$

$S_\infty = 6 \cdot \frac{4}{3}$

$S_\infty = \frac{24}{3}$

$S_\infty = 8$

Ex#1: Given the sequence 4, 7, 10, ... 13 16 19

- describe the pattern and write a *recursive* definition
- write an *explicit* definition
- find t_6 and t_{28}
- find S_6 and S_{28}

recursive

$$\begin{cases} t_1 = 4 \\ t_{n+1} = t_n + 3 \end{cases}$$

explicit:

$$t_n = 4 + 3(n-1)$$

$$t_6 = 19$$

$$t_{28} = 4 + 3(27)$$

$$t_{28} = 85$$

$$S_6 = \frac{6(4+19)}{2}$$

$$S_6 = 69$$

$$S_{28} = \frac{28(4+85)}{2}$$

$$S_{28} = 1246$$

Ex#2: Given the sequence 162, 54, 18, ...

- describe the pattern and write a *recursive* formula
- find an *explicit* formula
- find t_{12}
- find S_{12} and S

$\downarrow$

S_∞

Recursive:

$$\begin{cases} t_1 = 162 \\ t_{n+1} = t_n \cdot 1/3 \end{cases}$$

Explicit:

$$t_n = 162(1/3)^{n-1}$$

$$t_{12} = 162(1/3)^{11}$$

$$t_{12} = \frac{2}{2187}$$

$$S_{12} = 162\left(\frac{1-(1/3)^{12}}{1-1/3}\right)$$

$$S_{12} = \frac{531,448}{2187}$$

$$S_\infty = \frac{162}{1-1/3}$$

$$S_\infty = \frac{162}{2/3}$$

$$S_\infty = \overset{81}{\cancel{162}} \cdot \frac{3}{\cancel{2}}$$

$$S_\infty = 243$$

Ex#3: Given $t_{n+1} = 4 \cdot t_n$ ← Recursive - we must know where to start!

- list the first five terms (hmm... something's wrong?)
- find t_{12}
- find S_{12} and S

1, 4, 16, 64, . . .

3, 12, 48, . . .

$t_1 = 1$

$t_{12} = 1(4)^{12-1}$

$t_{12} = 4^{11}$

$S_{12} = \frac{1(4^{12}-1)}{4-1}$

$S_{12} = 5{,}592{,}405$

divergent $r > 1$

$\cancel{S_\infty = \frac{1}{1-4}}$

$\cancel{S_\infty = -\frac{1}{3}}$

NO!

Not possible

Ex#4: Given $t_n = 4n - 8$

- write a *recursive* formula
- find t_{15}
- find S_{15}

$t_1 = 4(1) - 8$

$t_1 = -4$

sub n+1 in for n

$t_{n+1} = 4(n+1) - 8$

$t_{n+1} = \underline{4n} + 4 - \underline{8}$

$t_{n+1} = t_n + 4$

$$\begin{cases} t_1 = -4 \\ t_{n+1} = t_n + 4 \end{cases}$$

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Assignment: page 695 #13, 17, 21, 25, 27, 32, 44, 45

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