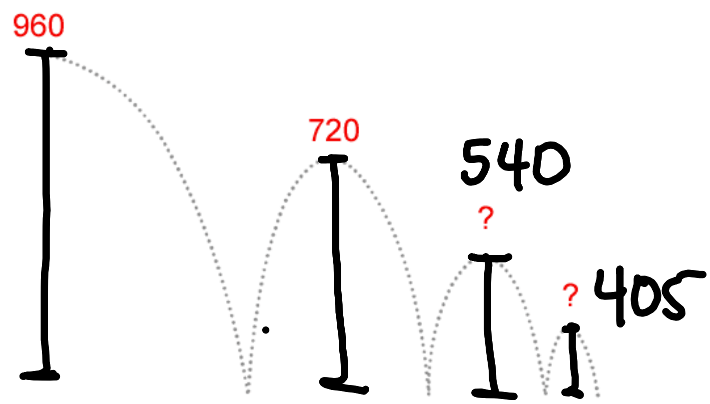
Ex#3: The heights form a geometric sequence...



$$\frac{720}{960} = \frac{3}{4}$$

Ex#4: QUICK! Find this sum

GEOMETRIC

$$1 + 2 + 4 + 8 + \ldots + 128 + 256$$

Explicit:

$a_n = a_1(r)^{n-1}$

$a_n = 1(2)^{n-1}$

a_n

1
2
4
8
16
32
64
128
256
511

$$S_n = \frac{a_1(r^n - 1)}{r - 1} \text{ OR } S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_9 = \frac{1(2^9 - 1)}{2 - 1}$$

sum of the first 9 terms.

$$S_9 = 511$$

$256 = 1(2)^{n-1}$

$\log_2 256 = n - 1$

$\log_2 2^8 = n - 1$

$8 = n - 1$

$9 = n$

We need a formula to sum a geometric series...

$$S_n = t_1 + t_1 r + t_1 r^2 + \ldots + t_1 r^{n-2} + t_1 r^{n-1}$$

Multiply both sides of the equation by r

$$r \cdot S_n = r \cdot \left(t_1 + t_1 r + t_1 r^2 + \ldots + t_1 r^{n-2} + t_1 r^{n-1} \right)$$

Subtract $S_n - r \cdot S_n$

$$S_n = t_1 + \cancel{t_1 \cdot r} + \cancel{t_1 \cdot r^2} + \ldots + \cancel{t_1 r^{n-1}}$$

$$- \quad r \cdot S_n = \cancel{t_1 \cdot r} + \cancel{t_1 \cdot r^2} + \ldots + \cancel{t_1 r^{n-1}} + t_1 \cdot r^n$$

$$S_n - r \cdot S_n = t_1 - t_1 \cdot r^n$$

$$\frac{S_n(1-r)}{1-r} = \frac{t_1(1-r^n)}{1-r}$$

$$\boxed{S_n = \frac{t_1(1-r^n)}{1-r}}$$

Ex#5: QUICK! Find the sum

$$54 + 18 + 6 + \overset{2 + \frac{2}{3} +}{\ldots} + \frac{2}{9}$$

$\swarrow$ 6th term

$$S_6 = \frac{54(1-(1/3)^6)}{1-1/3}$$

$$S_6 = \frac{54\left(1-\frac{1}{729}\right)}{2/3}$$

$$S_6 = \overset{27}{\cancel{54}} \cdot \frac{3}{\cancel{2}}\left(\frac{728}{729}\right)$$

$$S_6 = 3^4 \cdot \frac{728}{3^6}$$

$$S_6 = \frac{728}{3^2}$$

$$S_6 = \frac{728}{9}$$

Explicit:

$$t_n = 54(1/3)^{n-1}$$

$$\frac{2}{9} = 54(1/3)^{n-1}$$

$$3^2 \quad \frac{\overset{1}{\cancel{2}}}{9 \cdot \underset{27}{\cancel{54}}} \quad 3^3$$

$$\frac{1}{3^5} = \left(\frac{1}{3}\right)^{n-1}$$

$$\left(\frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^{n-1}$$

$$n = 6$$