HAT
Infinite Series

3/2/18

Warm Up:

- Given $\sum_{j=0}^{10}\left(j^4 - j\right) = 25{,}278$ find $\sum_{j=0}^{12}\left(j^4 - j\right)$

$25278 + 11^4 - 11 + 12^4 - 12$

60632

- Write using summation notation:

$$3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$$

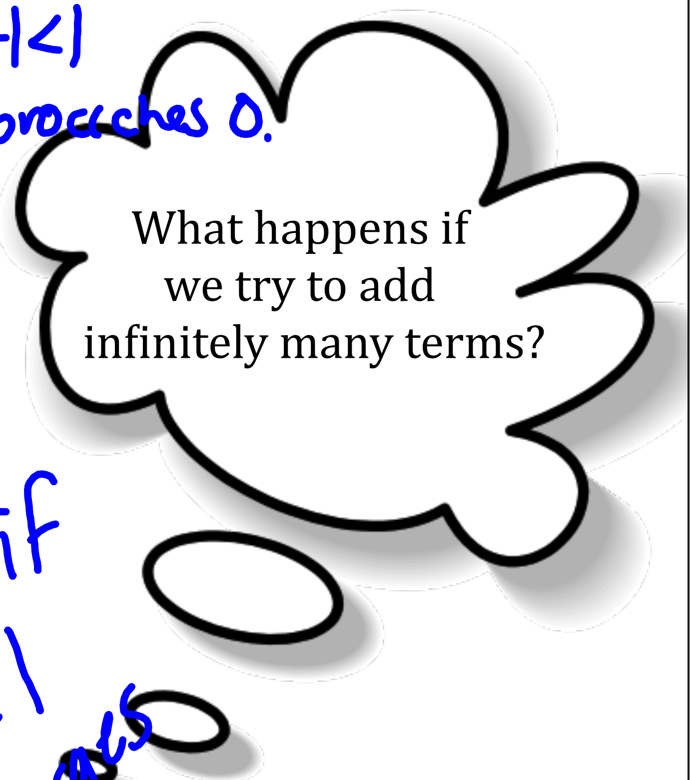
$\sum_{n=1}^{6} 3\left(\frac{1}{3}\right)^{n-1} \rightarrow S_6 = \frac{3\left(1-\left(1/3\right)^6\right)}{1-1/3}$

Ex#1: Estimate the sum $3+1+\frac{1}{3}+\frac{1}{9}+\ldots$

4.5

$$S_n = t_1\left(\frac{1-r^n}{1-r}\right)$$

if $|r|<1$ → approaches 0.



$$S_n = \frac{1}{1-r}\cdot t_1$$

$$S_n = \frac{t_1}{1-r}$$

only if $|r|<1$

converges

Ex#2: Given the sequence 40, 20, 10, ...

converging

- Find S_{10} $S_{10} = \frac{40(1-(1/2)^{10})}{1-1/2} \approx 79.9219$

Sum of the first 10 terms

- Find S_n $S_n = \frac{40}{1-\frac{1}{2}} = 80$

Find the infinite sum, all the terms

Ex#3: Find the sum of each infinite geometric series (if it exists).

- $-\frac{4}{3}+4+-12+\ldots$

diverges $r=-3$ no infinite sum

- $3-\frac{3}{2}+\frac{3}{4}-\frac{3}{8}+\ldots$

converges $r=-\frac{1}{2}$

$|r|<1$

$$S_n=\frac{3}{1-(-\frac{1}{2})}$$

$$S_n=\frac{3}{3/2}=2$$

Ex#4: Write $.\overline{27}$ as a fraction.

$.2727272727....$

$.27 + .0027 + .000027 + .00000027$

$r = \frac{1}{100}$ $\quad \frac{27}{100} + \frac{27}{10,000} + \frac{27}{1000000} + ...$

$S_n = \frac{27/100}{1 - 1/100}$

$S_n = \frac{27/100}{99/100}$

$S_n = 27/99$

$\boxed{S_n = 3/11}$

Assignment:

page 686 #17, 19, 21, 25, 27, 31, 33, 41, 47, 49, 52, 62, 65

March 2, 2018