# HAT
# Applications of Continuous Growth/Decay and Logistic Growth

12/8/17

#9. 100 mg – Initial  $\frac{1}{2}$ life = 4.47 billion years

How many years before there are 10mg left

$$50 = 100e^{k(4.47)}$$
$$\frac{1}{2} = e^{4.47k}$$
$$\ln\left(\frac{1}{2}\right) = \ln e^{4.47k}$$
$$\frac{\ln\left(\frac{1}{2}\right)}{4.47} = \frac{4.47k}{4.47}$$
$$-0.155066 = k$$

$$y = ae^{-0.155066\cdot t}$$
$$10 = 100e^{-0.155066\cdot t}$$
$$\frac{1}{10} = e^{-0.155066\cdot t}$$
$$\frac{\ln(1/10)}{-0.155066} = \frac{-0.155066\cdot t}{-0.155066}$$
14.849 **billion years** $= t$

$$y = a\left(\frac{1}{2}\right)^{t/4.47}$$
$$10 = 100\left(1/2\right)^{t/4.47}$$
$$\frac{1}{10} = \left(1/2\right)^{t/4.47}$$
$$\log_{1/2}(1/10) = \log_{1/2}(1/2)^{t/4.47}$$
$$4.47\cdot\log_{1/2}(1/10) = t/4.47\cdot 4.47$$
$$14.849 = t$$
billion years

8. Bone = 8000 yrs. old

$\frac{1}{2}$ life of Carbon-14 = 5730

$y = ae^{kt}$

$\frac{1}{2} = 1e^{k(5730)}$

$\frac{\ln(1/2)}{5730} = \frac{5730k}{5730}$

$-0.000121 = k$

$y = ae^{-0.000121 \cdot t}$

$y = 1e^{-0.000121(8000)}$

$y = 0.3799$

$y = 0.38$

38%

$y = a(1/2)^{t/5730}$

$y = 1(1/2)^{8000/5730}$

$y = .38$

Warm Up:

The half-life of Sodium-22 is 2.6 years. Write TWO decay equations for Sodium-22.

$$\boxed{y = ae^{-0.266595 \cdot t}} \qquad \boxed{y = a\left(\tfrac{1}{2}\right)^{t/2.6}}$$

$$\frac{1}{2} = 1e^{k(2.6)}$$

$$k = -0.266595$$

A geologist examines a meteorite and estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of Earth?

$$\frac{1}{10} = 1e^{-0.266595 \cdot t}$$

$$\frac{\ln(1/10)}{-0.266595} = \frac{-0.266595 \cdot t}{-0.266595}$$

$$8.637 = t$$

years

$$\frac{1}{10} = 1\left(\tfrac{1}{2}\right)^{t/2.6}$$

$$2.6 \cdot \log_{1/2}(1/10) = \frac{t}{2.6} \cdot 2.6$$

$$8.637 = t$$

yrs.

Ex#3:

The half-life of Carbon-14 is 5730 years. Find the equation for continuous decay for Carbon-14.

$y = a(1/2)^{t/5730}$

$k \approx \frac{\ln(1/2)}{5730}$

$y = ae^{-.000121t}$

Ex#4:

A specimen that originally contained 42 milligrams of Carbon-14 now contains 8 milligrams. How old is the fossil?

$$8 = 42e^{-0.000121 \cdot t}$$

$$\frac{8}{42} = e^{-0.000121 \cdot t}$$

$$\ln(8/42) = \ln e^{-0.000121 \cdot t}$$

$$\frac{\ln(8/42)}{-0.000121} = \frac{-0.000121 \cdot t}{-0.000121}$$

$$13{,}708.979 \text{ years} = t$$

$$8 = 42(1/2)^{t/5730}$$

$$\frac{8}{42} = \left(\frac{1}{2}\right)^{t/5730}$$

$$\log_{\frac{1}{2}}\left(\frac{8}{42}\right) = \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{t/5730}$$

$$\log_{\frac{1}{2}}\left(\frac{8}{42}\right) = \frac{t}{5730}$$

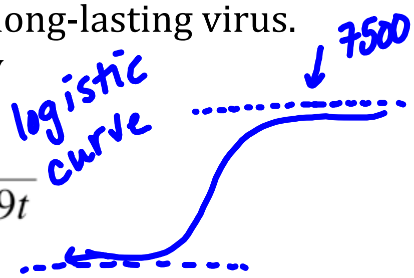
$$5730 \cdot \log_{\frac{1}{2}}\left(\frac{8}{42}\right) = t$$

$$13708.979 = t$$

Ex#1:

On a college campus of 7500 students, one student returns from vacation with a contagious and long-lasting virus. The spread of the virus is modeled by

$$y = \frac{7500}{1 + 7499e^{-0.9t}}$$




where $y$ is the total number of students affected after $t$ days. The college will cancel classes when 30% or more of the students are infected.

a) How many students will be infected after 4 days? $y = 36.425$ students

$t=4$

b) After how many days will the college cancel classes?

$.3(7500) = 2{,}250$

$1+7499e^{-.9t} \cdot 2250 = \frac{7500}{1+7499e^{-.9t}} \cdot 1+7499e^{-.9t}$

$1+7499e^{-.9t} = \frac{7500}{2250}$ $10/3$

$-1 \qquad -1$

$\frac{7499e^{-.9t}}{7499} = \frac{\frac{7}{3}}{7499}$

$\ln e^{-.9t} = \ln \frac{7}{3 \cdot 7499}$

$\frac{-.9 \cdot t}{-.9} = \frac{\ln\left(\frac{7}{3 \cdot 7499}\right)}{-.9}$

$t \approx 8.972$ days

Ex#2:
A city's population in millions is modeled by

$$f(t) = \frac{1.432}{1 + 1.05e^{-0.32t}}$$

where t is the number of years since 2000.

a. According to the function when will the city's population reach 1 million?

t

b. What will be the maximum population? 1.432 million

$$1 = \frac{1.432}{1 + 1.05e^{-0.32t}}$$

$$1 + 1.05e^{-0.32t} = 1.432$$

$$\frac{1.05e^{-0.32t}}{1.05} = \frac{.432}{1.05}$$

$$\ln e^{-0.32t} = \ln\left(\frac{.432}{1.05}\right)$$

$$\frac{-0.32t}{-0.32} = \frac{\ln\left(\frac{.432}{1.05}\right)}{-0.32}$$

$$t \approx 2.775$$

years