

HAT
Polynomial Long Division
and Synthetic Division

10/4/17

Warm Up: Simplify completely

$$\frac{(4x^5y^{-2})^3}{(2x^{-3}y)^0} = 4^3(x^5)^3(y^{-2})^3$$
$$= / = 64x^{15}y^{-6}$$
$$= \frac{64x^{15}}{y^6}$$

Today we are going to work on division of polynomials. Sometime we can factor and simplify to divide.

$$\text{Ex1: } \frac{5a^2b - 15ab^3 + 10a^3b^4}{5ab} \quad \frac{\cancel{5ab}(a - 3b^2 + 2a^2b^3)}{\cancel{5ab}}$$

$$a - 3b^2 + 2a^2b^3$$

$$\text{Ex2: } \frac{x^3 - 2x^2 - 15x}{x + 3}$$

$$\frac{x(x^2 - 2x - 15)}{x + 3} \rightarrow \frac{x(x - 5)\cancel{(x + 3)}}{\cancel{x + 3}} = \boxed{x^2 - 5x}$$

$$\text{Ex3: } \frac{x^3 + 9x^2 + 11x - 21}{x + 3}$$

Sometimes we need something else.

Ex3: Polynomial Long Division

$$\begin{array}{r}
 \underline{x+3} \overline{) x^3 + 9x^2 + 11x - 21} \\
 \underline{-(x^3 + 3x^2)} \\
 6x^2 + 11x \\
 \underline{-(6x^2 + 18x)} \\
 -7x - 21 \\
 \underline{-(-7x - 21)} \\
 0
 \end{array}$$

$x^2 + 6x - 7$ Quotient

$x^3 + 9x^2 + 11x - 21$
 $(x+3)(x^2 + 6x - 7)$
 $(x+3)(x-1)(x+7)$

No remainder
 $x+3$ is a factor
 of $x^3 + 9x^2 + 11x - 21$

Flashback!

$$\begin{array}{r} 135 \text{ R}10 \\ \underline{15} \overline{)2035} \\ \underline{-15} \downarrow \\ \quad 53 \\ \underline{-45} \\ \quad \quad 85 \\ \underline{-75} \\ \quad \quad \quad 10 \leftarrow \text{remainder} \end{array}$$

$135 \overset{10}{\div} 15$
 $135 \overset{2}{\div} 3$

Ex4: Use long division to find the quotient and remainder

Place
Holders

$$\begin{array}{r}
 3x^4 + x^3 - 2x + 1 \\
 \hline
 x^2 - 1
 \end{array}$$

$$\begin{array}{r}
 \underline{3x^2 + x + 3} + \frac{-x+4}{x^2-1}
 \end{array}$$

$$\begin{array}{r}
 x^2 + 0x - 1 \mid 3x^4 + x^3 + 0x^2 - 2x + 1 \\
 \underline{-(3x^4 + 0 - 3x^2)} \\
 x^3 + 3x^2 - 2x \\
 \underline{-(x^3 + 0 - x)} \\
 3x^2 - x + 1 \\
 \underline{-(3x^2 + 0x - 3)} \\
 -x + 4
 \end{array}$$

Remainder $\rightarrow -x + 4$

Ex5: Back to Ex3.

Complete this problem using synthetic division.

$$\begin{array}{r}
 \textcircled{x+3} \overline{) x^3 + 9x^2 + 11x - 21} \\
 \begin{array}{r}
 x+3=0 \\
 x=-3
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 -3 \overline{) 1 \quad 9 \quad 11 \quad -21} \\
 \textcircled{+} \downarrow \quad -3 \quad -18 \quad 21 \\
 \hline
 1 \quad 6 \quad -7 \quad \boxed{0}
 \end{array}$$

Remainder

$$\textcircled{x^2 + 6x - 7}$$

Ex6: Is $(2x-1)$ a factor of $f(x) = 2x^3 + 3x^2 - 8x + 3$?

YES!

leading coefficient $\neq 1$

$$2x-1=0$$

$$x = \frac{1}{2}$$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 3 & -8 & 3 \\ \oplus \downarrow & & 1 & 2 & -3 \end{array}$$

$$(2x-1)(2x^2 + 4x - 6)$$

$$\left(x - \frac{1}{2}\right)(2x^2 + 4x - 6)$$

$$\left(x - \frac{1}{2}\right) \cdot 2(x^2 + 2x - 3)$$

$$(2x-1)(x^2 + 2x - 3)$$

Ex7: Use synthetic substitution to find $f(-3)$ when

$$f(x) = 4x^4 - 3x^2 + 16x - 8$$

$$f(-3) = 4(-3)^4 - 3(-3)^2 + 16(-3) - 8$$

$$\begin{array}{r|rrrrr} -3 & 4 & 0 & -3 & +16 & -8 \end{array}$$

$$\begin{array}{r|rrrrr} \oplus & \downarrow & -12 & 36 & -99 & 249 \end{array}$$

$$\begin{array}{r|rrrrr} & 4 & -12 & 33 & -83 & \underline{241} \end{array}$$

$$f(-3) = 241$$

Assignment:

page 315 #15, 16
26, 27, 30 (Use long division)
20, 23, 24 (Use synthetic division)
36, 37, 39

